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*QUALIFICATION PROBLEM OF MATHEMATICAL MODELLING*

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*EVACUATION MODEL OF ISTANBUL SAPPHIRE TOWER*

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**CONTENTS:**

1. Summary.....	4
2. Introduction.....	5
3. Definition of The Problem.....	6
4. Clarification / Restatement.....	7
5. Assumptions	and
Variables.....	7
• General Assumptions.....	7
• Assumptions about the Residence.....	9
• Assumptions about the Shopping Mall.....	10
• Assumptions about the Underground Parking Lot .....	10
• Variables.....	11
6. Model.....	12
• Conceptual Model.....	12
○ Beginning.....	12
○ Trigonometry and Geometry.....	12
○ Arithmetic Sequence and Distance Covered by Stairs.....	13
○ Finding The Dimensions (Length, Height, Depth) of The Building.....	13
○ Finding The Dimensions of Shopping Mall.....	14
○ Finding The Number of People.....	14
■ Finding The Number of People In The Shopping Mall.....	14
■ Finding The Number of People In Apartments.....	15
• Mathematical Model.....	16
○ Modelling the Evacuation of People.....	16
■ Designing the Model.....	16
■ Collecting Data From Our Model.....	17
7. Solution of the Problem.....	18
a. Solution for the Residence.....	18

b.	<i>Solution for the Shopping Mall</i> .....	20
i)	<i>Formulating the Evacuation from the Shopping Mall</i> .....	20
c.	<i>Relation of Our Problem With Physics</i> .....	20
d.	<i>The Relation Between Capacitor Charging Function and the Evacuation</i> .....	21
e.	<i>The process of finding the C value</i> .....	23
f.	<i>Solving and Calculating the Integrals by Coding</i> .....	24
g.	<i>Finding the Time Result for the Shopping Mall</i> .....	27
h.	<i>Finding the Time Necessary for the Underground Parking Lot to Evacuate</i> .....	29
i.	<i>Finding the Time Result for Parking Lot</i> .....	34
8.	<i>Conclusion</i> .....	37
9.	<i>Reflection</i> .....	38
a.	<i>Strengths</i> .....	38
b.	<i>Limitations</i> .....	39
i.	<i>Limitations about the building</i> .....	39
ii.	<i>Limitations about the formulation</i> .....	40
10.	<i>Appendix</i> .....	42
a.	<i>Appendix 1.0</i> .....	42
b.	<i>Appendix 1.1</i> .....	43
c.	<i>Appendix 1.2</i> .....	44
d.	<i>Appendix 1.3</i> .....	45
e.	<i>Appendix 1.4</i> .....	46
f.	<i>Appendix 1.5</i> .....	47
i)	<i>Appendix 1.5.0</i> .....	47
ii)	<i>Appendix 1.5.1</i> .....	47
g.	<i>Appendix 1.6</i> .....	48
h.	<i>Appendix 1.7</i> .....	49
i.	<i>Appendix 1.8</i> .....	50
j.	<i>Appendix 1.9</i> .....	51

<i>k. Appendix 2.0</i> .....	52
<i>l. Appendix 2.1</i> .....	53
<i>m. Appendix 2.2</i> .....	54
<i>n. Appendix 2.3</i> .....	55
<i>o. Appendix 2.4</i> .....	56
<i>p. Appendix 2.5</i> .....	57
<i>q. Appendix 2.6</i> .....	58
<i>r. Appendix 2.7</i> .....	59
<i>s. Appendix 2.8</i> .....	60
<i>t. Appendix 2.9</i> .....	61
<i>u. Appendix 3.0</i> .....	62
<i>v. Appendix 3.1</i> .....	63
<i>w. Appendix 3.2</i> .....	64
<i>x. Appendix 3.3</i> .....	65
<i>y. Appendix 3.4</i> .....	66
<b>11. Bibliography</b> .....	<b>67</b>

**SUMMARY**

With the number of skyscrapers in major cities increasing, they have become an undeniable part of the human experience. A big part of this increase is that they are able to accommodate for a lot of people while not occupying a lot of surface area, due to their immense height. But accommodating these large sums of people becomes a real hassle during emergency situations. This is when the evacuation models come in. They aim to predict the evacuation time in order to take safety precautions. Without them the safety of these people would be at great risk. Our aim is to help make a tangible model for evacuations, so that the needed precautions can be taken for a smooth evacuation,

before it is too late. With our modelling we took Sapphire, the tallest building in Turkey, as a starting point.

In this paper, Sapphire was separated into three sections: the residence, the shopping mall and the underground parking lot. These sections were considered and modeled separately due to their unique conditions. The escape route of the evacuees was clarified and assumptions were made to make the model flow. After the necessary assumptions were made and data was collected, the modelling process was started from the residence by implementing the findings on a 3D simulation via the app “Blender”. A linear relationship between the number of evacuees and the taken for the residence was observed through the simulation. The relationship was then formulated into a linear graphical model. Due to the ratio of number of evacuees to number of floors being drastically different for the shopping mall and the underground parking lot, the conceptual models of these sections were based on an analogy of people being electrons that are escaping the capacitor which corresponds to the stairways. By mapping the given and estimated data to the parameters of the formula for charging of a capacitor, the time taken to evacuate the shopping mall and the underground parking lot were calculated separately. The evacuees of the underground parking lot were assumed to follow the evacuees of the shopping mall and the time taken for the last evacuee to leave the building was considered as the time taken for the evacuation of the whole building.

Through our research, we found out that it took between 2 to 2 and a half hours for a high building like Sapphire to collapse due to the fire. The result we obtained from our graphical models show that the evacuation of Sapphire takes 8 to 38 minutes, which is more than the critical time limit, while it should have lasted even a shorter time than 2 hours and is, therefore, an unsafe process. We proposed that Sapphire and other high buildings should consider adjusting their floor plans, capacities and escape routes in order to be able to safely evacuate the building within 2 to 2 and a half hours.

## ***INTRODUCTION***

With over-urbanization becoming prominent in major cities, comes an influx of larger and taller buildings. Thus, there is a newfound importance that building evacuation plans now have. In

order to spare casualties during emergencies, these evacuation plans need to be thoroughly thought out. Every single detail should be held as important while creating effective evacuation plans that minimize casualties. These plans also need to be made in such a way that allows the people in the building to exit before the situation becomes perilous. A mathematical model helps in predicting the time it may take the occupants to evacuate, so proper adjustments can be made.

These details consist of things, such as the distribution of people throughout the building, how this affects the staircases that are used, how the location of the hazard causes shifts in the planned route, and how the sudden flow of people in the stairways can delay the exit time. With this paper what we want to achieve is creating a model from scratch that helps with predicting the evacuation time for a high building as accurately as possible by considering the varying parameters.

### ***DEFINITION OF THE PROBLEM***

Hypothetically, a dangerous situation, which can be a fire or bomb notice, is reported in Sapphire, the tallest building in Turkey, at 2.30 pm on a weekday. Thereafter, the evacuation of people will be modelled, in which time depends on the number of people evacuated.

Sapphire comprises  $165,169 m^2$  total construction area, and with 10 floors beneath the ground floor, it has 56 floors above the ground level. Its antenna composes 30 meters of 261 meter total height, and a 4-floor public mall with a 6-floor parking lot is present in the underground construction of the Sapphire. The bearing system of the building is composed of reinforced concrete and steel; furthermore, basements and shopping center floors are planned as reinforced concrete, and the facade of the shopping mall as a continuation, the roof cover is steel construction. Ultimately, a total of 14 elevators, including 8 high speed ones, and 13 escalators are for vertical circulation; while, 8 moving walkways are for horizontal circulation.

Under the light of the given facts, assumptions will be made, and modelling will be created via using 3-D and 2-D structure to express the most relevant relation between the variables.

### ***CLARIFICATION / RESTATEMENT***

The model will be made with the intentions of estimating the evacuation time in Sapphire, which is made up of 3 different sections that will be dissected and calculated in a mutually exclusive way. These sections are:

- 1) The residence
- 2) The mall
- 3) The underground parking lot

After the initial calculations are done separately, the final calculations will be done with the aspects of the building in mind.

### ***ASSUMPTIONS AND VARIABLES:***

#### ***General assumptions:***

- For the flow of the model, it is assumed that the structure of the building will hold up and there won't be any disorders caused by the fire in the stairways that are considered safe (one stairway will be eliminated because it won't be a safe path as the definition of the problem suggests).
- Since the people on the ground floor would have direct access to the way out, the time it takes for them to escape would be negligible. Due to this, it is assumed that the people in the ground floor won't cause a delay in time and the number of people on the ground floor will be taken as 0 in the simulation since the number of people on the ground floor won't have an effect on the outcome.
- All occupants in the building are assumed to have identical bodies; their height, shoulder width, and speed are based on studies that show average body ratios and speed of humans.

- There are 10 floors below the ground level: the top 4 make up the mall and the bottom 6 make up the underground parking lot.
- The average shoulder width of a human is taken as 0.5 m. (*Classic Jacket Sizing*)
- The average height of a human is taken as 1.695 m (The mean of average height of women and average height of men was taken to obtain this result). (*İşte Türkiye'deki ortalama erkek ve kadın boyu*)
- It is assumed that the mass of an average human is 72.9 kg(the mean of average mass of women and average mass of men in Turkey) (*Türkiye Kilo Ortalamaları*)
- It is assumed that people have a constant speed of 1.4 m/s for the 3D simulation (*Tempolu Bir Yürüyüş Hızı Ne Kadar Hızlıdır?*) and that people are not affected by psychological factors that hinder their performances in terms of running or following the escape route.
- It is assumed that the air resistance does not have an impact on people's motion; air resistance, in general, is neglected.
- It is assumed that the usage of escalators, elevators and moving walkways during evacuation is not possible and that no one was in between two floors when the fire started.
- According to the blueprints of the building, there are two stairways in the residence that go all the way down to the -10th floor but their entrances are closed by surrounding walls in the shopping mall section of the building, and it is assumed that only the residents of the residence have access to their entrances from the underground parking lot. There are two other stairways that start at the ground floor and go all the way to the -10th floor. Since the modelling of the residence is made under the assumption that all occupants of the residence are initially in the residence, it is assumed that the people in the shopping mall and the underground parking lot are only capable of using the two stairways that are separate from the residence's stairs. (*İstanbul Sapphire*)
- It can be seen from the blueprints of the residence that an occupant who lives on one side of the floor can't reach the stairs on the opposite side of the floor via doors or hallways. Thus,



for the evacuees that come from the residence to have an escape route, the burning stairway should be one of the separate stairways that take place in between floors 0 to -10. Therefore, the modelling will be made under the assumption that one of the stairways which takes place between floors 0 to -10 cannot be used during evacuation. (*Istanbul Sapphire*)

- When creating a 3D simulation and calculating the dimensions of the building, the antenna is disregarded and the height of the building is taken to be 231 m (*Istanbul Sapphire*)

### ***Assumptions about the residence:***

- It is assumed that all apartments are fully occupied.
- There are 56 floors above ground level that make up the residence and it is assumed that people are distributed equally throughout the residence and every floor except the ground floor(*due to the reasons mentioned in general assumptions*) is occupied by people in the beginning.
- According to the number of beds in the apartment blueprints of the residence (the larger beds were counted as two people and the smaller beds were counted as one person), the average number of people to live on each floor was estimated to be 22. (*Istanbul Sapphire*) (Birer, Emel, Doç. Dr.)
- Both stairways of the residence are open to access during evacuation due to the reasons stated in the general assumptions. It is also assumed that the occupants have been given an escape route that equally distributes them to each stairway meaning, the occupants that live in the middle sections of the floor and are able to access both sides will divide into two equal groups.
- Due to lack of information given on the amount of people in mechanical floors and the observation deck that takes place in the top 2 floors, the estimated value of 22 occupants per floor is assumed to be valid for these floors. Also, sticking to the assumption that the occupants are distributed equally among the floors; the average number of people per floor would turn out to be more accurate if the estimations are based on the apartment floors which

take up 48 out of 56 floors. (*İstanbul Sapphire*) For these reasons, the model of the residence will be made under the assumption that there are 22 people per floor.

### ***Assumptions about the shopping mall:***

- People in the mall are assumed to be distributed equally among the floors. They are also assumed to be distributed equally throughout their floor.
- Due to lack of information on the number of people in the shopping mall, we had to develop a logical method to estimate the amount of evacuees that come from the mall. Since the shopping mall is a crowded area, it is assumed that people have their own invisible, spherical bubbles whose diameters are their arm spans and that the number of people on a floor is equal to the number of bubbles that can fit on the floor. According to the body proportions in Leonardo Da Vinci's Vitruvius Man, the average height of humans is taken to be equal to the average arm span (*The Vitruvian Man - by Leonardo da Vinci*). Therefore the diameter of the invisible bubbles are taken to be 1.695 m. A geometric approach was made to find how many bubbles can fit to the surface area of a floor of the shopping mall. The obtained value for the amount of people per floor is 2597. Thus, the total number of people in the shopping mall is assumed to be 10388.

### ***Assumptions about the underground parking lot:***

- It is assumed that the surface area of the floors of the underground parking lot is equal to that of the mall as seen in the blueprints of the building (*İstanbul Sapphire*)
- According to our research, the number of people per  $m^2$  in underground parking lots are taken to be 0.033 (*Çakıcı YÜKSEK BİNALARDA ACİL BOŞALTIM SÜRESİNİN BELİRLENMESİ*). Assuming that the area of one floor of the underground parking lot is  $61.19 \times 121.92 = 7460.2848 m^2$  (this calculation is based on findings about the dimensions of the building, which will be further explained in this paper) the number of people in each

floor of the underground parking lot is  $7460.2848 \times 0.033 \approx 246$  and the total number of people in the parking lot is  $246 \times 6 = 1476$ .

**Variables:**

**Table 1.0: The variables used in graphical modelling of the problem**

<b>Symbol</b>	<b>Variable</b>
$n$	<i>This value indicates the number of floors to be evacuated</i>
$x$	<i>This value represents the total number of evacuees and is the input of the model</i>
$y$	<i>This value indicates the time taken by the defined amount of people to evacuate. It is the output of the model.</i>

**Table 1.1: Parameters used in graphical modelling of the problem**

<b>Symbol</b>	<b>Value</b>	<b>How it is Calculated</b>
$L$	9.2 m	<i>Total distance covered by the stairs on one floor</i>
$A$	2.04 m <sup>2</sup>	<i>width of staircase × average length of a human (1.2 × 1.695)</i>
$R$	$\frac{L \times n}{A}$	<i><u>Total distance covered by the stairs on one floor × number of floors</u> width of staircase × average length of a human</i>
$C_m$	205.44606	<i>This value represents capacitance for the mall section of</i>

		<i>the building. It is a stable parameter that represents resistance caused by different limiting conditions that will further be discussed in this paper</i>
$C_o$	4.244893	<i>This value represents the capacitance of the underground parking lot and will also be further discussed in this paper</i>

## **MODEL**

### **CONCEPTUAL MODEL**

- **Beginning**

After being done with our assumptions and variables, we began to model the simulation of people evacuating by considering what we had found so far. Our first aim was to figure out how to implement our finding on a 3D Model simulating program. Therefore, we started to learn how the application *Blender* works in order to make the evacuating animation happen.

- **Trigonometry and Geometry**

While we were learning the fundamentals of the application “Blender” , we applied our trigonometry knowledge and skills to build up the model of stairs of Sapphire. After that by considering a blueprint of Sapphire with scales, we found out the real corresponding lengths (**Figure 1.7**) of the stairs. So based on our trigonometric calculations, we have found that every stair has a height of 2.0625 meters and has a length of 2.6 meters. Also by dividing the number of floors by the height of the building (  $231 \div 56$  ), we obtained the value 4.125 meters. Since the height of the stairs are 2.0625 which is the exact half value of 4.125, we concluded that there are two identical stairs at each floor. Also, there are three square plates which have a length and width of 1.2 meters. The first plate that comes up is in front of the first stairs. Second plate is between the first and second stairs.

The last plate comes after the second stairs. This diagram shows the geometrical design of stairs and plates on a 2d plane. (**Appendix 1.0**)

- **Arithmetic Sequence and Distance Covered by Stairs**

By considering that, we found the total distance that will be taken by all people. It can be done by adding up the length of 2 stairs and 3 plates. Since the length of one plate is 1.2 meters and there are 3 of them, since the length of the one stair is 2.6 meters and there are 2 of them, we added up all those values  $1.2 \times 3 + 2.6 \times 2$  and we obtained the value 9.2 meters. This means that every floor covers a distance of 9.2 meters. By considering the arithmetic sequence formula, we gathered up the values and obtained this formula:  $u_n = 9.2 + 9.2 \times (n - 1) = 9.2 \times n$ . In this formula, the value  $n$  refers to the floor number. As the the floor number increases one by one, the total distance that stairs and plates cover from the floor  $n$  to the bottom, increases by 9.2 meters. In addition, the highest floor number in Sapphire is 56. Thus, by multiplying 56 and 9.2 (  $9.2 \times 56$  ), we obtained the total distance covered by the stairs from all the way from the top to the bottom as 515.12 meters.

- **Finding The Dimensions (Length, Height, Depth) of The Building**

We knew that in order to find the close value of people at all the floors of the shopping mall, we had figured out the approximate area of the shopping mall and come up with a result. With that being said, we started by comparing dimensions of Sapphire with its own blueprint. Since we already knew that the height of the building is 231 meters, we compared its height value to its blueprint. (**Appendix 1.1**) By considering our calculations, 231 meter heights corresponds to 10.8 centimeters. Also the depth of the building corresponds to 1.5 centimeters. (**Appendix 1.2**) So we applied a linear ratio to find the depth of the building in terms of meters. We found the depth as 32.08 meters. However, the blueprint that we used as a reference was showing only the depth and height of the building. Since it was a 2d blueprint (**Appendix 1.1**), there was no room for the third dimension. Therefore, we figured out a way to find the length of the building by considering the area and number of apartments. The smallest apartment had an area of 120 meter squares and the largest apartment had

an area of 1100 meter squares. As we could not learn about how apartments differ in terms of area covered, we took the arithmetic mean of the smallest and biggest apartments. We found the mean value as 610 meter squares. (**Appendix 1.2**). We also learned that - based on our searches- there were totally 177 apartments in the building. Because of this information that we found, we multiplied those two values (610 meter squares and 177) to find out how much area apartments do cover approximately. So we obtained the total area covered by apartments as 107610 meter squares. (**Appendix 1.2**). To find out the area covered by apartments per floor, we divided what we have just found to 55 which is the floor number of the building without considering the ground-base floor since there is no apartment block at ground level. (**Appendix 1.2**) So, we obtained the total area covered by apartments per floor as 1963 meter squares. (**Appendix 1.2**). By considering the properties of a rectangle, the product of depth and length of the building should be equivalent to 1963 meter squares, since 1963 meter squares is just the area covered per floor, not by the whole building. So by dividing the area covered by apartments per floor (1928 meter square) by depth of the building (32.08 meter) , we found the length of the building as 61.19 meters. (**Appendix 1.2**)

- **Finding The Dimensions of Shopping Mall**

By considering the blueprint of the building, we can state that the shopping mall has a much larger dimension than the building has. So just like what we did, again we compared the blueprint values with real world values and from there we found that the shopping mole has a depth of 121.92 meters and has a length of 61.19 meters. (**Appendix 1.3**)

- **Finding The Number of People**

- a) **Finding The Number of People In The Shopping Mall**

In this case, we took a geometrical approach to approximate the possible number of people who can be in the shopping mall. By considering Vitruvius Man ,which was made by Leonardo Da Vinci, the length of the path from our one finger to the other finger is equal to our height. Before moving on we searched up for the average height in Turkey and found it as 1.695 meters. So If a

person tilts both of their arms with a 90 degree angle, this person forms up an invisible circle with a radius of exactly half of their height. Although we put forth the circle and radius idea, we imagined people at the shopping mall as squares because just like people in there, the shopping mall has a rectangular shape. **(Appendix 1.4)** So when we divide the shopping mall area by a single person's area, we got a better approximation. Therefore, instead of assuming a person as a circle and dividing that one circle which is in this case one person's area, to the shopping mall's area, we assume people in the shopping mall as squares. Thus, we found the total area covered by a single person as the square of their height. **(Appendix 1.4)** After finding a single person's area as their heights square, we divided what we have found to the shopping mall's area which was 61.19 m times 121.92 m. (  $121.92 \text{ meter} \times 61.19 \text{ meter}$  ) At the end we obtained the value 2597. **(Appendix 1.4)** This is the number of people per shopping mall's floor. Since we know that the shopping mall has 4 floors under the ground level, we multiplied 2597 with 4. Then we got the value 10388 which is the total number of people in the shopping mall. **(Appendix 1.4)**

#### **b) Finding The Number of People In Apartments**

In order to find out the average number of people per floor, we aimed to calculate the average bed number per floor. Most of the cases, there were 12,10,9 beds. **(Appendix 1.5.0)** Those values varied on every floor and because of that we did not know how they were ordered and how they differ in terms of amount and number. In order to get ourselves through this, we took their arithmetic mean:  $\frac{12+10+9}{3}$ . We also learned that every bed was supposed to contain 2 people. So after multiplying the arithmetic mean of bed numbers by 2, we obtained the value 20.7. Since we did not have access to the exact number of people on every floor, we obtained a non-integer value. So after finding the value as 20.7, we rounded it to 22. The reason behind rounding the value to 22, not 21 is because we were going to divide the number of the people into half since there were 2 identical stairs. **(Appendix 1.5.0)**. As we know 21 can not be divided into half. The closest integer to 21 that can be divided into half is 22. This was the reason why we considered 22 as the people number per floor. Since there were 56 floors, from  $56 \times 22$ , we got 1232 as the total number of people. Even Though there are 22

people at the ground floor we were not going to include them since those people at the ground floor can go out without using stairs. Because of that they would not cause any decrease delay.

**c) Finding the number of the people in parking slot**

The method of finding the total people value was explained in detail in the assumptions section.

## ***MATHEMATICAL MODEL***

- **Modelling the Evacuation of People**

- **Designing the Model**

We began to model by designing the outside look of our stair model. It had to look proportional from outside and also from the inside so that it could provide us the most accurate data points possible. Firstly, we implemented the geometrical properties of the stairs to our model on the modelling application called *Blender*. Because of the technical difficulties and high hardware requirements of the application, we were only able to model the first 4 stairs of the building. **(Appendix 1.6)** In addition, there were some differences between the building's real stairs and our model's stairs. In terms of length and height, geometrically, they were an exact replica of the real building's stairs. However, there were some minor differences. First of all, we had to take all people as identical spheres in order to make them able to rotate, move and turn, since a person's fast walking speed is approximately 1.4 m/s. In addition those spheres had a radius of half of an average person's shoulder width which is 25 cm. Since one sphere -person- had a width of 50 cm and the width of stairs were 1.2 m, two spheres were able to fit in one row. Moreover, those identical spheres -people- had a mass of 72.9 kg which is the mean of the average male and female masses in Turkey. Since those spheres do not have an artificial intelligence backing them up, they were going to move by only initial velocity and gravitational acceleration. Because of this fact, we had to create a half circle looking shape by putting triangles in 15 degree descending orders. By creating the shape that acts like a half



circle, the spheres would not lose their kinetic energy so much to the point where they stop, because that half circle shape smoothly changing curves decrease the kinetic energy loss by reducing the impact of collisions. (**Appendix 1.8**) Moreover, we added some bumpers in some critical places to avoid spheres bouncing and jumping factors because of collisions with walls and other spheres. (**Appendix 1.9**) Since in real life evacuation there would be no jumping or bouncing, those additions were necessary to make our collected data more precise and accurate. Last but not least, we added stairways that stand as the path that connect apartments and actual stairs. (**Appendix 2.3**)

- **Collecting Data From Our Model**

After completing the design of our model, we started to collect data from our graph. After every parameter was adjusted correctly, we started the animation. Since we assumed that there were 22 people at every floor and there were two identical stairs, at every floor there were 11 spheres. (**Appendix 2.0**) The application calculates the time in terms of frames and 1 second is equivalent to 25 frames. Every sphere is a little bit higher than the position of stairways. It takes approximately 30 frames for each sphere to land down on the stairways. (**Appendix 2.1**) This means whenever we calculated the taken time, before we converted frame value to in seconds, firstly we subtracted 30 frames from the taken frame value since in real life people won't be landing on stairways from a very little higher place that causes 30 frames to be wasted. After collecting all our datas. We found out that there is a linear relationship. By considering our findings, (**Appendix 2.2**) we observed that when the 11th sphere runs down just one stairway from the first floor, it takes 16.4 seconds. When the 22nd sphere runs down just one stairway from the second floor, it takes 17.2 seconds. When the 33rd sphere runs down just one stairway from the third floor ,it takes 16.2 seconds. When the 44th sphere runs down just one stairway from the fourth floor, it takes 16.6 seconds. (**Appendix 2.2**) Another very important detail that we observed was the spheres that are located at different floors did not collide with spheres that are located on different floors. For example the spheres that started to run down from the second floor, did not interact or collide with the spheres that started to run down from the first floor, third floor, or fourth floor. This meant that only the spheres that were located at the same

floor number, collided, interacted with each other. They did not bother the spheres that were from other floors. As shown in here, spheres that are from different floor numbers, do not meet, interact or collide. (Appendix 2.4)

## ***SOLUTION OF THE PROBLEM***

### **a) Solution for the Residence**

As far as we observed, the 11th sphere completes one staircase in 16.4 seconds, 22th sphere completes one staircase in 17.2 seconds, 33th sphere completes one staircase in 16.2 seconds, 44th sphere completes one staircase in 16.6 seconds, Later on, we decided to take the arithmetic mean of all those taken time values to be more precise. *Arithmetic mean* =  $\frac{16.4 + 17.2 + 16.2 + 16.6}{4}$

= 16.6 seconds. Since we knew that the evacuation process progresses linearly because of the fact that spheres that are located at different floor numbers do not meet, we can use linear ratio to find out how much of the 16.6 seconds goes to complete stairways and how much of the 16.6 seconds goes for to complete one staircase. Since the total distance is 10 + 9.2 - the stairway has a length of 10 meters and as explained early on, the stairs that connect other floors with each other have a length of 9.2 meters. - 16.6 seconds are necessary to complete a 19.2 meters long path. In addition, theoretically, since we adjusted the human speed as 1.4 m/s, 19.2 meter over 16.6 seconds fraction ( $\frac{19.2\ m}{16.6\ s}$ ) should have given 1.4 m/s value to us but did not, it gave us 1.16 m/s. However, there was a reason for that. Even though the spheres don't meet the other spheres that are on different floors, spheres meet and collide with the spheres that are next to them, initially. Because of this fact some short ranged delays - like 1-2 seconds - occurred and that caused the average speed to decrease. Later on we considered the new speed that our model provided to us ( $\frac{19.2\ m}{16.6\ s} = 1.16\ m/s$ ), not the one that  $\frac{\text{meter}}{\text{seconds}} = 1.4\ m/s$  because in the real evacuation situation those small delays and decrease in average speeds would occur. So, we took delays in consideration with the new results. The key point was that we obtained a linear relationship. If it takes 16.6 seconds for 19.2 meters, it takes 7.8 seconds

for 9.2 meters to complete stairs and if it takes 16.6 seconds for 19.2 meters, it takes 8.5 seconds for 10 meters to complete stairways. We obtained those values by using linear ratio. (**Appendix 2.2**) By considering the arithmetic formula:  $9.2 \times n$ ,  $n = \text{number of floors}$ , and the necessary time which is 7.8 seconds for 9.2 meter distance; we saw that the time would become  $7.8 \times n$ . However, we had to consider the fact that the first floor which is the ground floor has direct access to gates which enables them to escape in nearly no time. This meant that the occupants that are initially on the ground floor would be neglected because they don't contribute to the outcome and that all the people that start to evacuate from the other 55 floors would be losing 7.8 seconds more to run out from the building, since they have to arrive to the ground floor in order to reach to the gates and it takes 9.2 meters more for people to complete. (**Appendix 2.6**) An additional 9.2 meters means an additional 7.8 seconds because of the linear ratio mentioned before. Due to these results, we used a scatter plot to plot our datas and found a linear function ( $y = mx + n$ ) (**Appendix 2.5**). However we changed the function from  $0.70901 \times (x) + 16.3$  to  $0.70901 \times (x - 11) + 16.3$ . The reason was that even though we neglected the people on the ground floor, we counted the ground floor's distance so we had to formulate the model according to the 56-floor plan. In order to eliminate the delay that would have been caused by the occupants on the ground floor if it wasn't neglected, we decided to include the total amount of evacuees regarding all 56 floors as input values and subtract 11 (the number of people from the ground floor that would correspond to a stairway) from it before multiplying it by the coefficient of the input. So by changing the function in this way, when we plug in the value half of the number of all people which is 616 since we divided people into half while they were evacuating because of the fact that there were two identical stairs, it would automatically decrease 11 people by neglecting the ground floor. So the changes in function were on point. Therefore as an answer, we found out that the time that it takes to empty the residence (apartments) is:

$$f(x) = 0.70901 \times (x - 11) + 16.3 = y \text{ seconds}$$

$$f(56) = 0.70901 \times (616 - 11) + 16.3 = 445.3 \text{ seconds} \approx 7.42 \text{ minutes} \approx 7 \text{ minutes } 25 \text{ seconds}$$

, where x is the half of the total people in the apartments.

## **b) Solution for the Shopping Mall**

### **i) Formulating the Evacuation from the Shopping Mall:**

Based on what we have found so far, there are 2597 people on each shopping mall floor. Since there are 4 shopping mall floors under the ground level, from  $4 \times 2597$ , we got 10388 people. We found out that the number was much larger compared to the total number of people in whole apartments which was 1210 since the surface area of the shopping mall is larger and the shopping mall is supposed to be more crowded because it is open to public use. Because of the massive difference in the number of people, we stated that delays in speed and time and collisions between people would be significantly larger. Because of that we were supposed to find out and use an exponential function that increases with the evacuated people along with a decreasing slope because of the high human concentration formation at the stairs and the occurrence of much larger new delays in time and human speed. Since there was not a specific function that has been already developed for our problem, we had to connect this problem to a specific field in physics by considering the same working principles to form up a function for our problem.

### **c) Relation of our Problem with Physics**

Firstly we made research about electrons, flow of electrons through wires, charging & discharging capacitors with electrons because of the similarity in concepts. Later on we recognized some tangible similarities. Just like electrons running through wires to reach the positive side (+) from the negative side (-), people were running through stairs to reach the exit from the floor they had started on. (**Appendix 2.7**) Although the proportions between electrons and people and the proportions between the wire that electrons run through and the stairs that people run through, were visibly and theoretically different, the concept and progress were the same. In addition, there is also one other major aspect that real life evacuation and electron flow have in common and it is the resistance. In wires, there is a resistance value that has an impact on electrons flow rate. This resistance value has a directly proportional relationship with the length of the wire and has an inversely proportional relationship with the cross section area of the wire. If we increase the length of

the wire that electrons flow through, it will become harder for electrons to reach the negative side from the positive side since the distance is now longer. So the resistance of the wire would increase. However, if we increase the cross section area of the wire that electrons flow through, the number of electrons per row would increase. So the rate of flow would increase. Because of that if we increase the cross section area of the wire, the resistance value of the wire would decrease. (**Appendix 2.9**) By considering those relationships. We got this formula that describes resistance in terms of length and cross section area. (**Appendix 2.8**) There was also a another parameter called resistivity and shown as “ $p$ ” in the formula which indicated the resistive force applied by the material the wire was made up of when the electrons passed through. In our case, the resistive force that people would encounter when travelling in staircases would have been air resistance. However, since we neglected the effect of air resistance on people’s performances and speed, we neglected the value of this parameter as well. Therefore, the value “ $p$ ” was not used in the graphical modelling.

#### **d) The Relation Between Capacitor Charging Function and the Evacuation**

After verifying the relations, connections between electrons running processes through wireless and exacting process of people through stairs, we digged deeper surrounding the topic. Even though we related our problem with a specific field of physics, we still did not have a function to describe the evacuation process of people from shopping malls. We continued our research and came across the capacitor charging function on the internet. We learned about a circuit equipment called a capacitor. (**Appendix 3.0**). The function of a capacitor is to store electrons by charging itself up with electrons and discharge them at some point. This was very critical information since stairs in the building also acted very similarly. Stairs allow people to come inside the building just like capacitors that allow the electrons to run through the inside. Also the process of people emptying the building by going through the stairs is just like electrons that run through the capacitor to charge & discharge it. So based on our findings that were about those similarities, as our attempt to formulate the problem, we considered every floor's staircases as capacitors. We tried to apply the capacitor charging function to our real world (**Appendix 3.2**) situation. We took all details and similarities into consideration and

that was what we got (**Appendix 3.1**). We obtained values for the corresponding variables and parameters of the function, which are listed in the *assumptions and variables*. After finding nearly every corresponding real life of capacitor charging function (C value would be obtained further down in the steps), we worked on the resistance value of every staircase. Since the length of the staircases changes by  $9.2 \times n$  meters per floor considering our arithmetic sequence for the stair length, we found out that at the  $n^{th}$  shopping mall floor, the length of the stairs would become  $9.2 \times n$ . So we began to find the cross section area of the stairs. We already knew the width of all the stairs is 1.2 meters. We also knew that the average height of people living in Turkey, was 1.695 meters. We knew that If we multiply those two values, we would obtain a cross section area of the stairs. (**Appendix 3.3**) Thus, we obtained the formula as this. From  $\frac{\text{length of stairs}}{\text{cross sectional area}}$ , we

got  $\frac{9.2 \times n}{1.2 \times 1.695}$  where n is the number of the shopping mall's floor. After we were done with

finding a general form for the resistance of stairs. We started to develop our function. By considering what we found as function properties and similarities (**Appendix 3.1**), we stated our function would

going to look like this  $2597 \times (1 - e^{\frac{-t}{R \times C}})$  where R is  $\frac{9.2 \times n}{1.2 \times 1.695}$  and

where n is the floor number of the shopping mall. Also when there is an increase in the floor number the resistance will also increase because of the directly proportional relationship between length and resistance . If the resistance increases per floor, it would take longer for people whose floor number is higher. We also formulated what we just stated in this sheet. (**Appendix 3.4**) However this function takes time (in seconds) as input and gives an output which is the number of people that evacuated the building until the given time in the input is up. We wanted the exact opposite because our independent variable was the number of people and from there we wanted to find out the necessary time for evacuation, which makes the time variable automatically a dependent variable. Therefore, by considering function transformation rules, we found the inverse function of that which was

$$\left(-R \times C\right) \times \left(\ln\left(-\frac{x-2597}{2597}\right)\right) \quad \text{where } R \text{ is } \frac{9.2 \times n}{1.2 \times 1.695} \text{ and where } n \text{ is the floor}$$

number of the shopping mall. This final function gets the number of people under the variable  $x$  and gives us the time that it would take for the  $x$ 'th person to get out from the building. However there was still something missing about the function: the  $C$  value. Since there was no real life corresponding value for  $C$  we had, because of the lack of reliable information about the buildings architectural and the number of small details in the architecture of the building, we knew that we had to keep the  $C$  value constant because it would not be right to change it like variables  $R$  when it does not have any real life corresponding value. The second reason why we kept the value  $C$  constant was that capacitors are made in factories with a given constant value (in farad) which means  $C$  is expected to be constant. So, as we assumed the stairs as capacitors, the capacitance of the stairs should be also constant. Moreover, the definition of capacitance suggests that "*Capacitance is a measure of a non-conducting material's ability to store energy by creating a separation of charge across a potential difference (voltage).*" (*Dusto Capacitance: Definition, Formula & Units*) Meaning that it corresponds to a property of the capacitor and in our case, the stairway. Since the structure or material properties of the staircase itself won't change throughout the evacuation as we assumed, the value of  $C$  would stay constant. Because of these reasons we decided to keep the  $C$  value constant.

- **The process of finding the  $C$  value**

Firstly, before we move on, let's take a look at the function that we had found earlier which was for the residence evacuation:  $f(x) = \frac{7.8}{11} \times (x-1) + 16.3$ . We also concluded that the exponential function which was  $\left(-R \times C\right) \times \left(\ln\left(-\frac{x-2597}{2597}\right)\right)$  should look and act as similar as possible for the first shopping mall floor compared to  $f(x)$  since as we have shown in here (**Appendix 3.4**), the first floor should be evacuated faster than the 2nd, 3rd and 4th shopping mall floors because of the resistance value. Even though we stated that this formula should look and act similar for the first shopping mall floor compared to  $f(x)$ , unlike the  $f(x)$ , this formula was a logarithmic function. Thus, there was an obvious degree difference between functions. Because of this very important fact, instead of observing to what extent that logarithmic function for the first shopping

mall floor looks and acts similar compared to  $f(x)$  manually, we used integral to calculate the area under both of those functions from the 0 to 2597. Since 2597 was the total number of people per shopping mall floor, there was no need to calculate the further area. If the area difference of those functions from 0 to 2597 gets closer to 0, the logarithmic function would become as close and similar as possible to  $f(x)$ . In addition, the value that would be making the area difference between them, was  $C$  since it was the only remaining parameter. That was our strategy to find the  $C$  value. We were going to look up for the value that makes the area difference between those two functions very close to 0 and this value would become our  $C$  value.

- **Solving and Calculating the Integrals by Coding**

So far we've had these two integrals:

$$\int_0^{2597} ((-R \times C) \times (\ln(-\frac{x-2597}{2597}))) dx) : \text{The integral of}$$

exponential shopping mall function

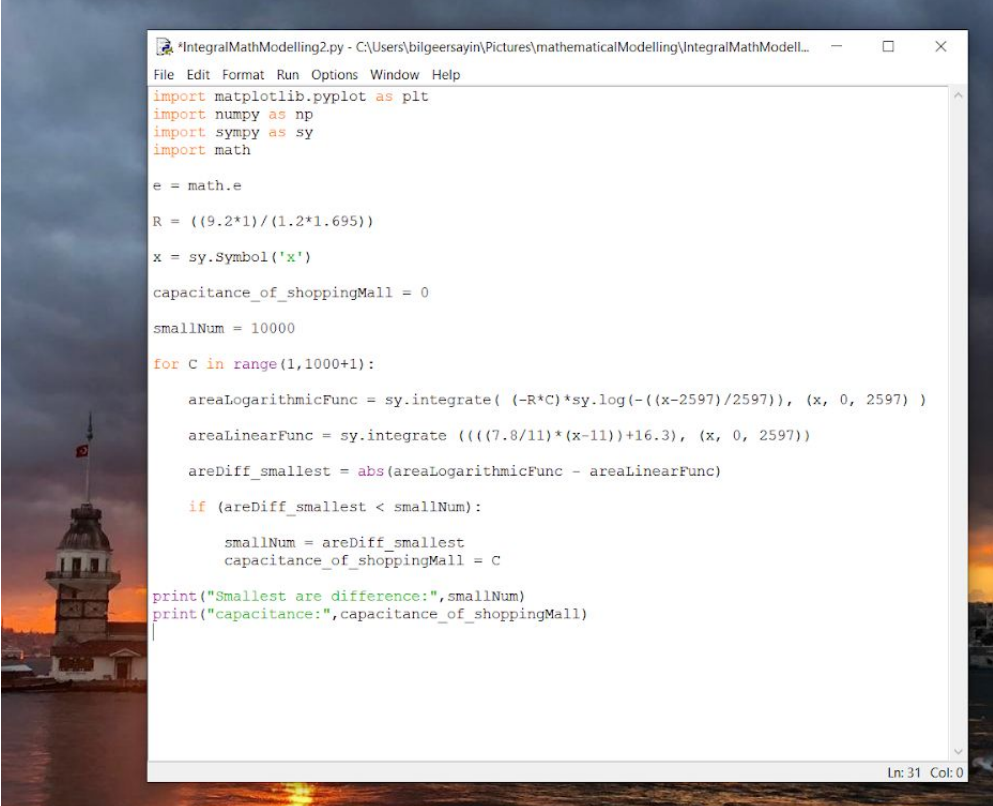
$$\int_0^{2597} ((\frac{7.8}{11} \times (x - 1) + 16.3) dx) : \text{The integral of the } f(x)$$

function

After mathematically defining what our two integrals were going to be, it came down to finding the best suitable  $C$  value. However, the program that we were going to code, would only find the **integer** value for  $C$ . Because making the program find best **decimals** along with the **integer** value, would be very inefficient in terms of algorithm complexity. So our plan was to write a program that finds the **integer** part of the suitable  $C$  value. Then we planned to find best decimal places to add to the integer part of  $C$  by manually playing with the decimals to make the difference of the areas



under the curves even more precise and closer to 0. By considering our plan, that was the program that we coded (**Figure 1.1**)



```
IntegralMathModelling2.py - C:\Users\bilgeersayin\Pictures\mathematicalModelling\IntegralMathModell...
File Edit Format Run Options Window Help
import matplotlib.pyplot as plt
import numpy as np
import sympy as sy
import math

e = math.e

R = ((9.2*1)/(1.2*1.695))

x = sy.Symbol('x')

capacitance_of_shoppingMall = 0

smallNum = 10000

for C in range(1,1000+1):

    areaLogarithmicFunc = sy.integrate( (-R*C)*sy.log(-(x-2597)/2597), (x, 0, 2597) )

    areaLinearFunc = sy.integrate (((7.8/11)*(x-11))+16.3), (x, 0, 2597))

    areDiff_smallest = abs(areaLogarithmicFunc - areaLinearFunc)

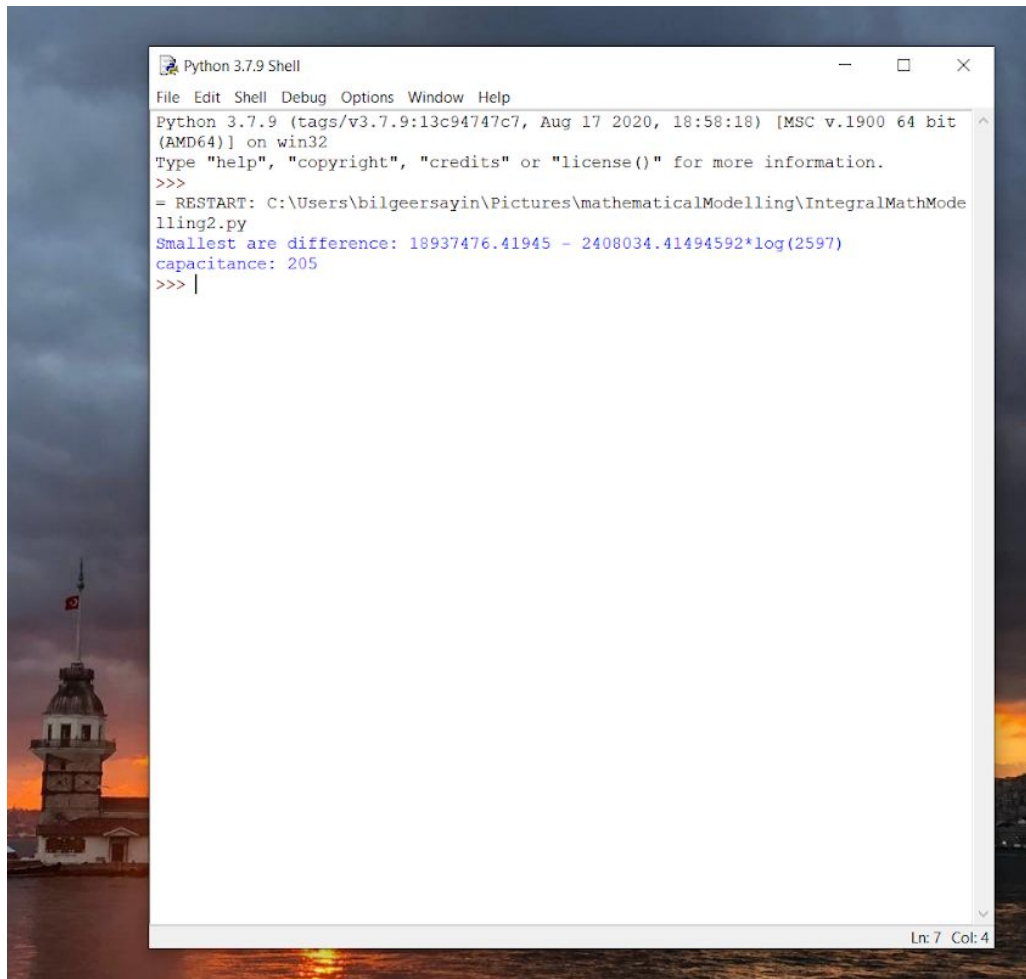
    if (areDiff_smallest < smallNum):

        smallNum = areDiff_smallest
        capacitance_of_shoppingMall = C

print("Smallest are difference:",smallNum)
print("capacitance:",capacitance_of_shoppingMall)

Ln: 31 Col: 0
```

For solving the problem, firstly we defined the constant values like the e and R values. We defined R value as  $\frac{9.2 \times 1}{1.2 \times 1.695}$  because we were only considering the shopping mall's first floor. After that we defined a loop for the C value. At that loop, the program tries all **integer** values between 1 and 1000 (1 and 1000 included) to find the best suitable value for C since it is not possible to calculate an integral of a function that contains a different unknown parameter than the one and only x. We created this loop because our goal was to make the integral difference (area difference) at least smaller than 10000. If a C value provides a smaller area difference than 10000, it becomes the C value temporarily. The reason it lasts temporarily is that if a different value for C provides even just a little bit of a smaller area difference value than the previous value, It becomes the new C value. We made this algorithm to work for values between 1 and 1000 (1 and 1000 included) as we said. At the end, the program found the best suitable C **integer** value as 205.



```
Python 3.7.9 Shell
File Edit Shell Debug Options Window Help
Python 3.7.9 (tags/v3.7.9:13c94747c7, Aug 17 2020, 18:58:18) [MSC v.1900 64 bit
(AMD64)] on win32
Type "help", "copyright", "credits" or "license()" for more information.
>>>
= RESTART: C:\Users\bilgeersayin\Pictures\mathematicalModelling\IntegralMathMode
lling2.py
Smallest are difference: 18937476.41945 - 2408034.41494592*log(2597)
capacitance: 205
>>> |
```

After finding out the **integer** value for C which was 205, we manually changed the decimals of 205 and obtained the value 205.44606 which was nearly perfect since it made the area difference smaller than 0.009 .(**Figure 1.2**)

1  $C_1 = 205.44606$

2  $\frac{7.8}{11} \cdot (x - 11) + 16.3$

3  $(-R_1 \cdot C_1) \left( \ln \left( -\frac{x - 2597}{2597} \right) \right)$

4  $\text{abs} \left( \left( \int_0^{2597} (-R_1 \cdot C_1) \left( \ln \left( -\frac{x - 2597}{2597} \right) \right) dx \right) \right)$   
 $= 0.00835719937459$

1  $C_1 = 205.44606$

2  $\frac{7.8}{11} \cdot (x - 11) + 16.3$

3  $(-R_1 \cdot C_1) \left( \ln \left( -\frac{x - 2597}{2597} \right) \right)$

4  $\left. \right) dx \left. \right) - \left( \int_0^{2597} \left( \frac{7.8}{11} \cdot (x - 11) + 16.3 \right) dx \right) \left. \right) \left. \right)$   
 $= 0.00835719937459$

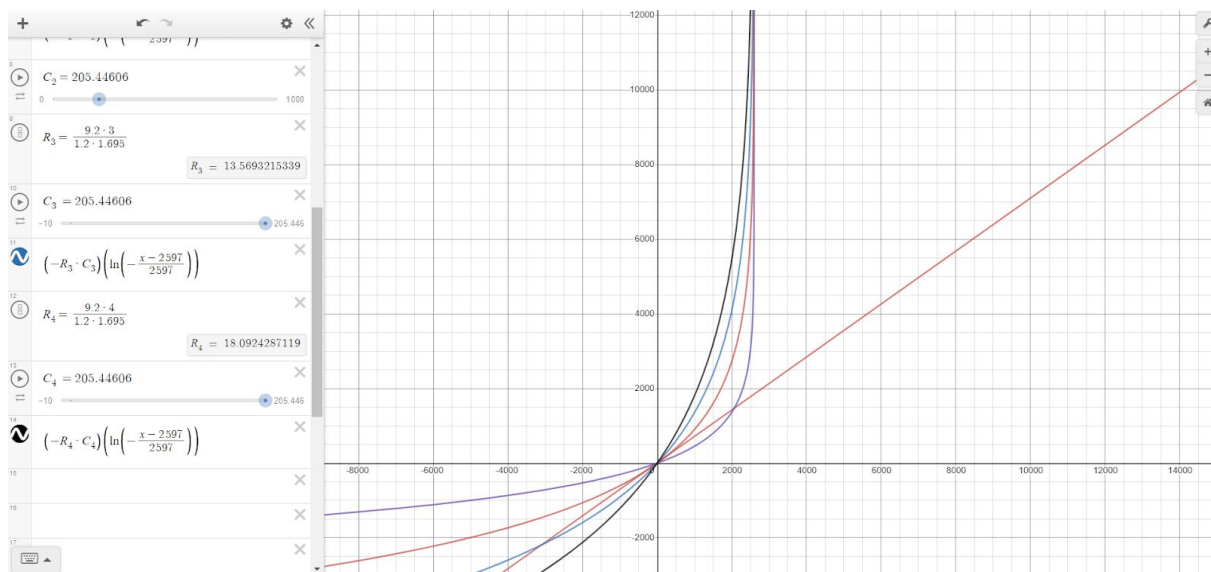
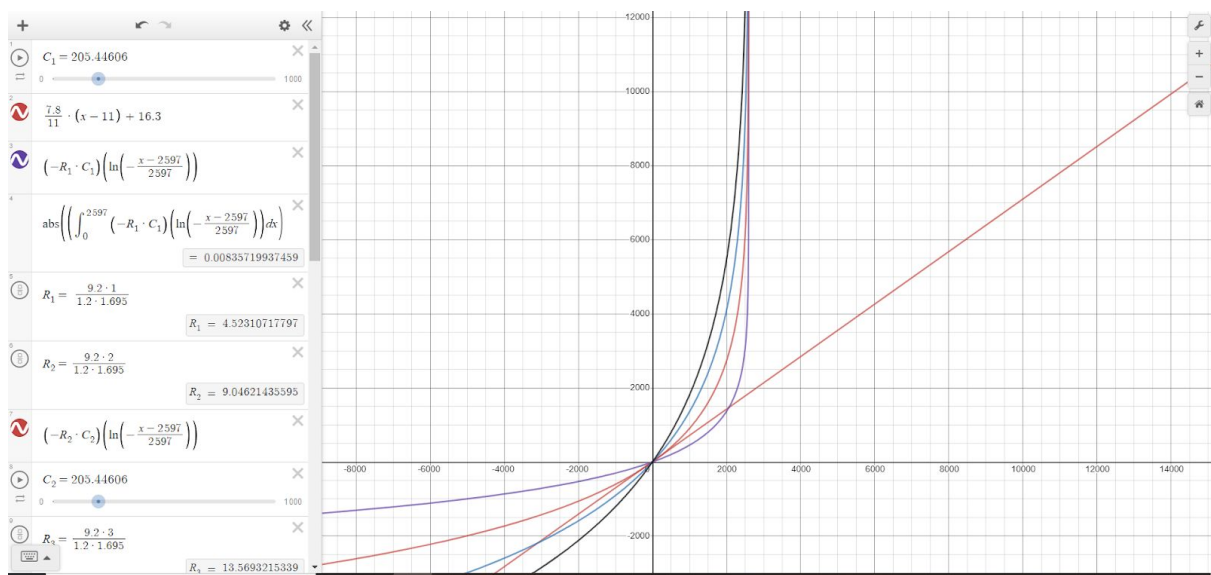
- **Finding the Time Result for the Shopping Mall**

After finding a suitable value for C. We changed R values according to the floor numbers by considering our finding for R  $\frac{9.2 \times n}{1.2 \times 1.695}$  where n is the shopping

mall's floor number. For example the second floor's resistance was  $\frac{9.2 \times 2}{1.2 \times 1.695}$ , the third floor's

resistance was  $\frac{9.2 \times 3}{1.2 \times 1.695}$  and lastly the fourth floor's resistance was  $\frac{9.2 \times 4}{1.2 \times 1.695}$ , So we

applied our findings to graphs and gathered those results and graphs. (Figure 1.3)



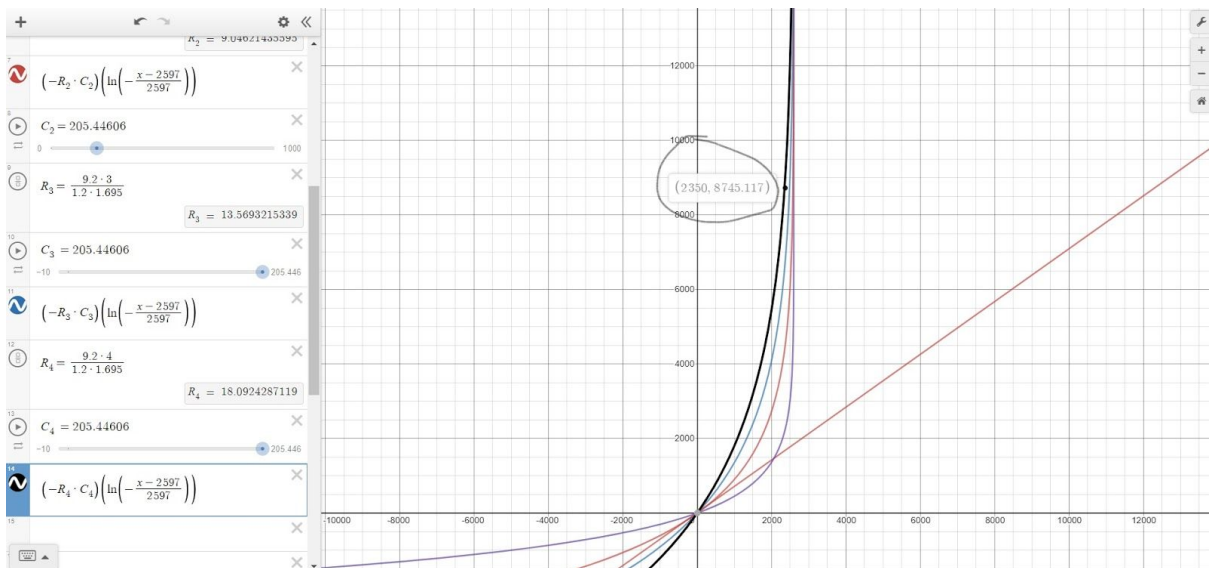
The black colored function which is the function stands for the people at the 4th floor of the shopping mall, increases with the smallest slope since it has the most stair resistance because of this R formula:

$$\frac{9.2 \times n}{1.2 \times 1.695}$$

where n is the shopping mall's floor number. So we should only consider black

colored function since when the last several people were ready to evacuate from the building, the

people at 1st floor, 2nd floor and 3rd floor of the shopping mall would have already evacuated the building. Because of this we only considered the necessary time for the black colored function which is the 4th floor. However, all of these logarithmic functions had a vertical asymptote of 2597. We had to make a wise decision on choosing the right last person in order to have accurate results from the graph. When the values pass the people number passes 2400 the necessary time gets unrealistically bigger because of the function properties of logarithmic functions. By considering this, we tried to choose the biggest people number as possible in order to be accurate. We selected the 2350th person as a reference. By considering 2350th person as our reference point, we found out the necessary time to evacuate the shopping mall for approximately 8745 seconds. **(Figure 1.4)**



The result for shopping mall  $\approx 8745 \text{ seconds} \approx 2.43 \text{ hours} \approx 2 \text{ hours}, 26 \text{ minutes}$

- **Finding the Time Necessary for the Underground Parking Lot to Evacuate**

We applied all the same procedures for finding the time for auto parks evacuations. Since auto park floors start from -5 and go all the way to -10. The starting resistance value for the first floor of the auto park would be  $\frac{9.2 \times 5}{1.2 \times 1.695}$ . So again we had to find the C value for the parking lot by

coding a program that works again with the same method and algorithm. Since the people number per parking lot was 246, we changed the logarithmic function and created a new logarithmic function for parking lots. The new function that we created was  $(-R \times C) \times (\ln(-\frac{x-246}{246}))$  where

R is  $\frac{9.2 \times n}{1.2 \times 1.695}$  and where n is the floor number of the park lot. Again, we were going to make the

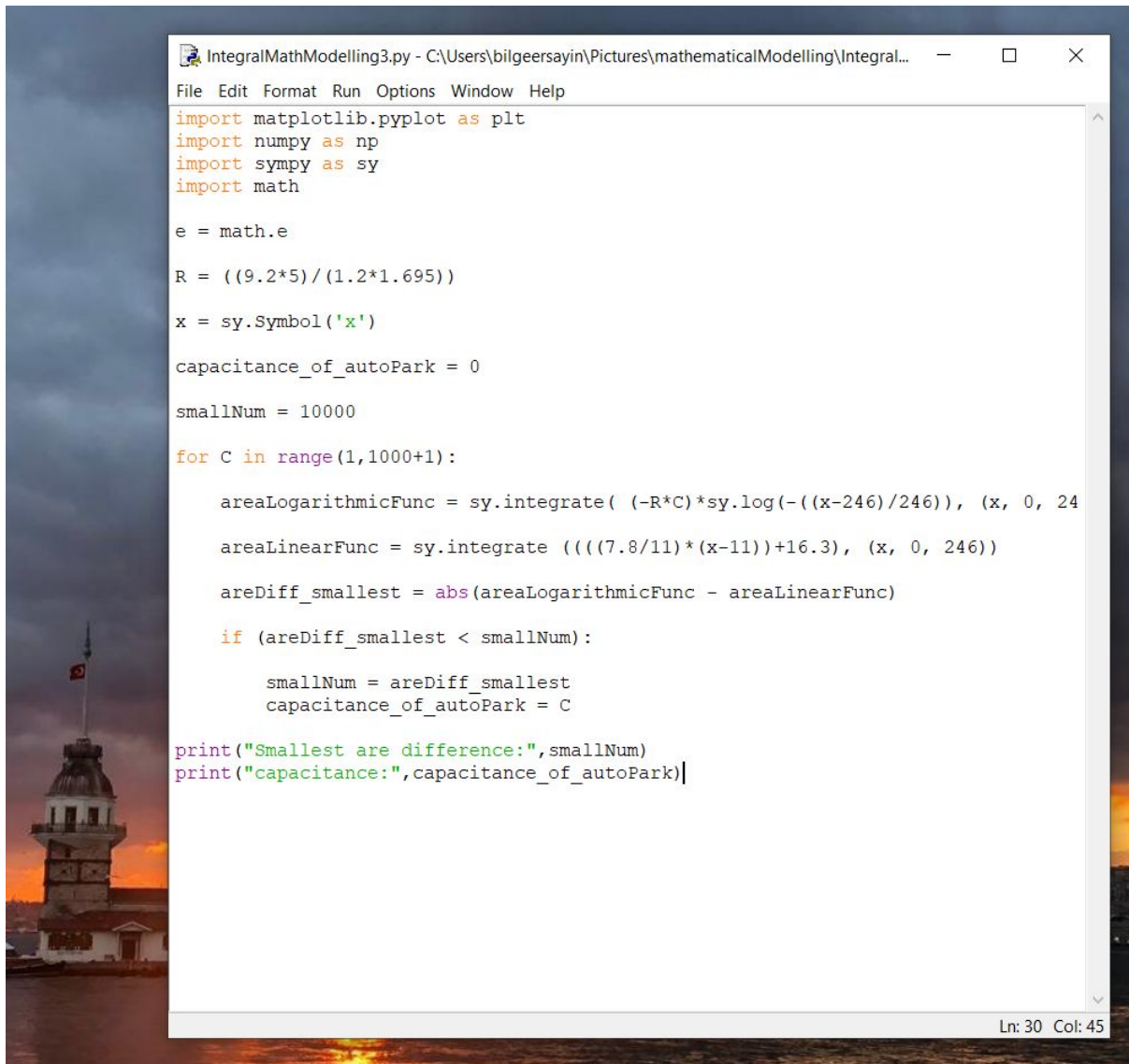
area difference near to 0 in order to make the parking lot's first floor function as close and similar as possible to the f(x) function. Creating a new logarithmic function lead us to a change in the function's integral and the new integral value became this:

$$\int_0^{246} ((-R \times C) \times (\ln(-\frac{x-246}{246}))) dx$$

The integral of the f(x) function was also changed because of the different people's numbers.

$$\int_0^{246} ((\frac{7.8}{11} \times (x - 1) + 16.3)) dx$$

Again we wrote a program to find out the **integer** part of the best possible value of C for the park lot. However, the program that we wrote, found only the **integer** value for C. Because making a program to find best **decimals** while finding the **integer** value, would be very sterile and poor in terms of algorithm complexity. Thus, our goal was to create a program that finds the **integer** part of the suitable C value. Then we planned to find best decimal places to add to the **integer** part of C by manually playing with the decimals to make the difference of the areas under the curves even more precise and closer to 0. By considering our plan, that was the program that we coded (**Figure 1.5**)



```
IntegralMathModelling3.py - C:\Users\bilgeersayin\Pictures\mathematicalModelling\Integral...
File Edit Format Run Options Window Help
import matplotlib.pyplot as plt
import numpy as np
import sympy as sy
import math

e = math.e

R = ((9.2*5)/(1.2*1.695))

x = sy.Symbol('x')

capacitance_of_autoPark = 0

smallNum = 10000

for C in range(1,1000+1):

    areaLogarithmicFunc = sy.integrate( (-R*C)*sy.log(-(x-246)/246)), (x, 0, 246)
    areaLinearFunc = sy.integrate (((7.8/11)*(x-11))+16.3), (x, 0, 246))

    areDiff_smallest = abs(areaLogarithmicFunc - areaLinearFunc)

    if (areDiff_smallest < smallNum):

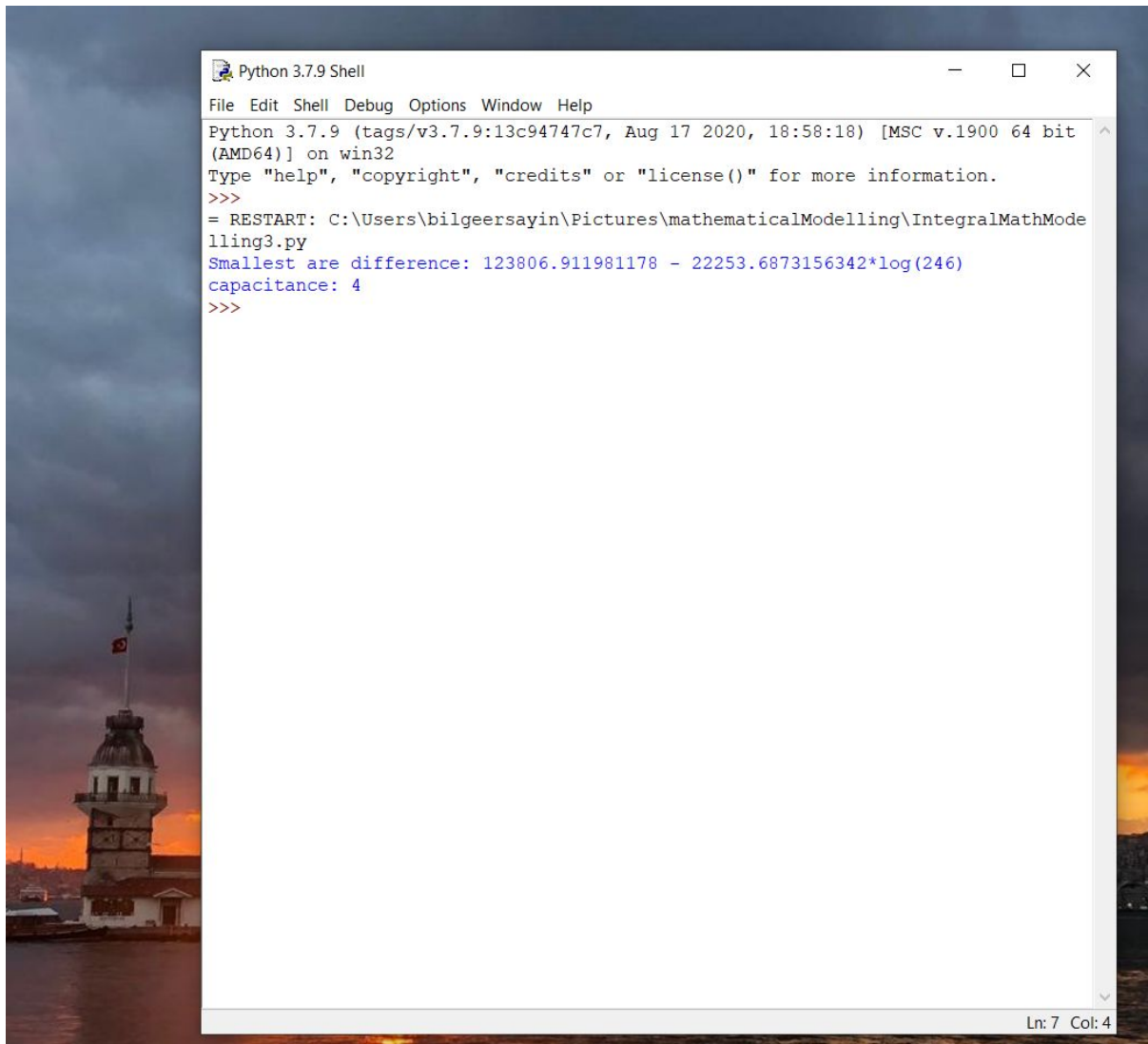
        smallNum = areDiff_smallest
        capacitance_of_autoPark = C

print("Smallest are difference:",smallNum)
print("capacitance:",capacitance_of_autoPark)
```

Ln: 30 Col: 45

For solving the problem, firstly we defined the constant values like the e and R values. We defined R value as  $\frac{9.2 \times 5}{1.2 \times 1.695}$  because we only considered the first floor of the park lot which is the -5th floor that comes after the -4th of the shopping mall.. After that we defined a loop for the C value. At that loop, the program tries all **integer** values between 1 and 1000 (1 and 1000 included) to find the best suitable value for C since it is not possible to calculate an integral of a function that contains a different unknown parameter than the one and only x. We created this loop because our goal was to make the integral difference (area difference) at least smaller than 10000. If a C value provides a smaller area difference than 10000, it becomes the C value temporarily. The reason it lasts temporarily is that if a different value for C provides even just a little bit of a smaller area difference







value than the previous value, It becomes the new C value. We made this algorithm to work for values between 1 and 1000 (1 and 1000 included) as we said. At the end, the program found the best suitable C integer value as 4.



```
Python 3.7.9 Shell
File Edit Shell Debug Options Window Help
Python 3.7.9 (tags/v3.7.9:13c94747c7, Aug 17 2020, 18:58:18) [MSC v.1900 64 bit
(AMD64)] on win32
Type "help", "copyright", "credits" or "license()" for more information.
>>>
= RESTART: C:\Users\bilgeersayin\Pictures\mathematicalModelling\IntegralMathMode
lling3.py
Smallest are difference: 123806.911981178 - 22253.6873156342*log(246)
capacitance: 4
>>>
```

After finding out the integer value for C which was 4, we manually changed the decimals of 4 and obtained the value 4.244893 which was nearly perfect since it provided us with an area difference less than 0.002 .(Figure 1.6)



1	 $\frac{7.8}{11} \cdot x + 8.5$	✕
2	 $C_1 = 4.244893$  0  6	✕
3	 $(-R_1 \cdot C_1) \left( \ln \left( -\frac{x-246}{246} \right) \right)$	✕
4	 $R_1 = \frac{9.2 \cdot 5}{1.2 \cdot 1.7}$ <div style="border: 1px solid gray; padding: 2px; display: inline-block;"><math>R_1 = 22.5490196078</math></div>	✕
5	$\text{abs} \left( \left( \int_0^{246} (-R_1 \cdot C_1) \left( \ln \left( -\frac{x-246}{246} \right) \right) dx \right) - \right)$ <div style="border: 1px solid gray; padding: 2px; display: inline-block;"><math>= 0.00155668450316</math></div>	✕
6		✕

1  $\frac{7.8}{11} \cdot (x - 11) + 16.3$

2  $C_1 = 4.244893$

3  $(-R_1 \cdot C_1) \left( \ln \left( -\frac{x - 246}{246} \right) \right)$

4  $R_1 = \frac{9.2 \cdot 5}{1.2 \cdot 1.7}$   
 $R_1 = 22.5490196078$

5  $\left. \right) dx \left. \right) - \left( \int_0^{246} \left( \frac{7.8}{11} \cdot (x - 11) + 16.3 \right) dx \right)$   
 $= 0.00155668450316$

- **Finding the Time Result for the Parking Lot:**

After finding a suitable value for C. We changed R values according to the floor numbers by considering our the formula for R  $\frac{9.2 \times n}{1.2 \times 1.695}$  where n is the shopping mall's floor number. For

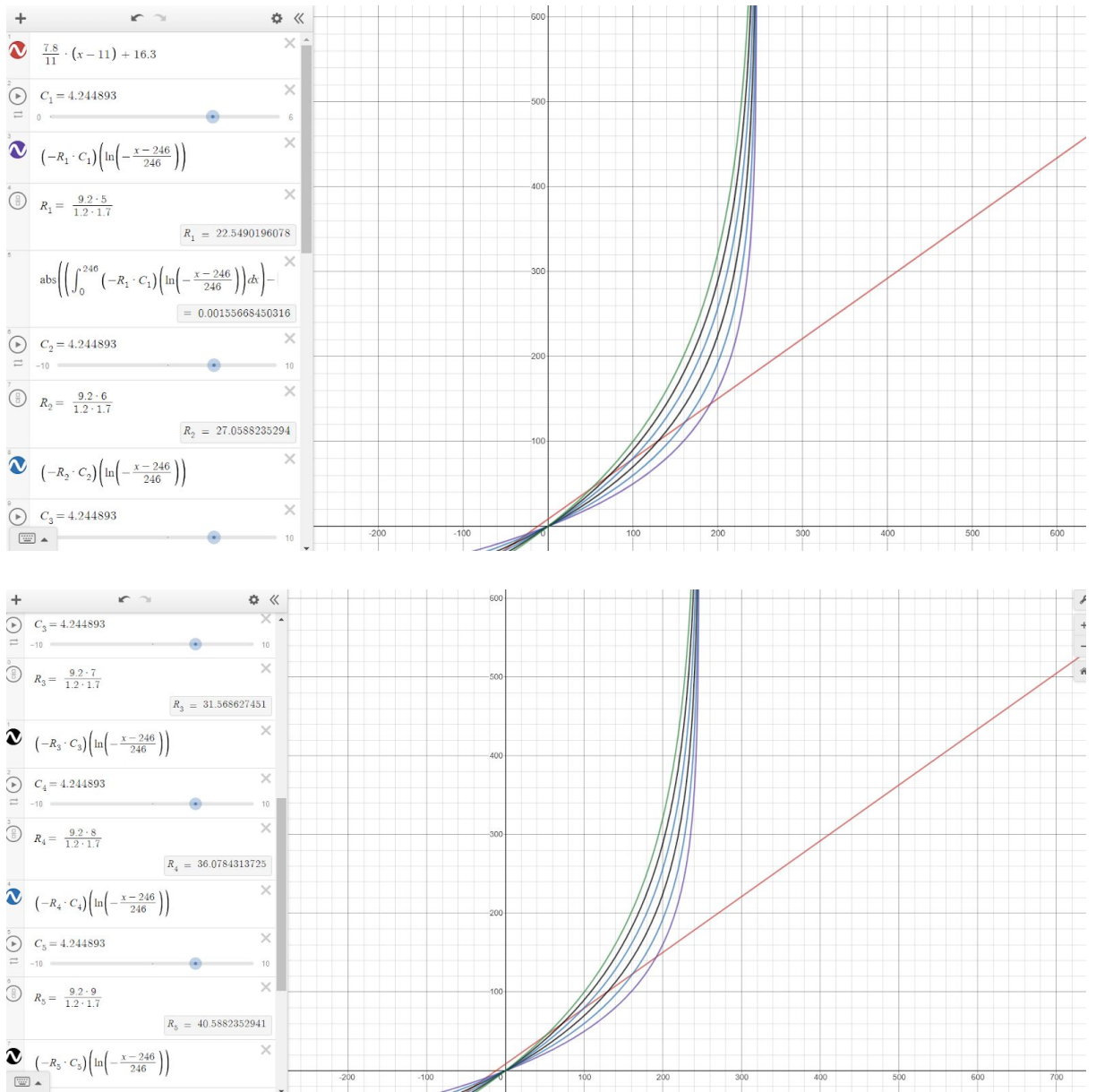
example the second floor's resistance (-6th floor) was  $\frac{9.2 \times 6}{1.2 \times 1.695}$  , the third floor's resistance

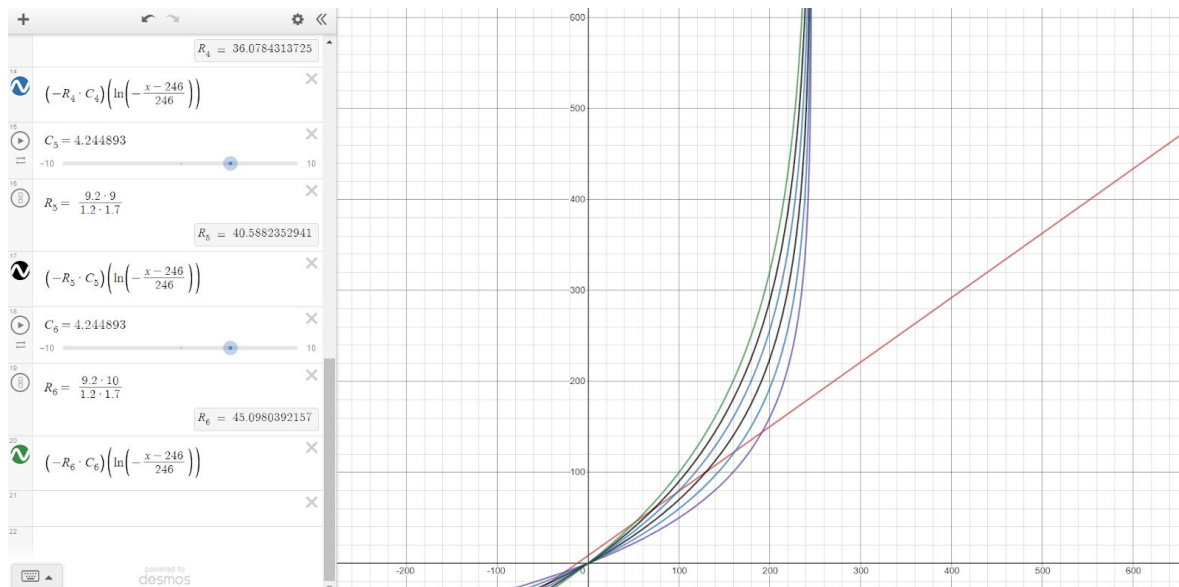
(-7th) was  $\frac{9.2 \times 7}{1.2 \times 1.695}$  , the fourth floor's resistance (-8th) was  $\frac{9.2 \times 8}{1.2 \times 1.695}$  , the fifth floor's

resistance (-9th floor) was  $\frac{9.2 \times 9}{1.2 \times 1.695}$  ,the sixth floor's resistance (-10th floor) was

$\frac{9.2 \times 10}{1.2 \times 1.695}$  , So we applied our findings to graphs and gathered those results and graphs.

(Figure 1.7)

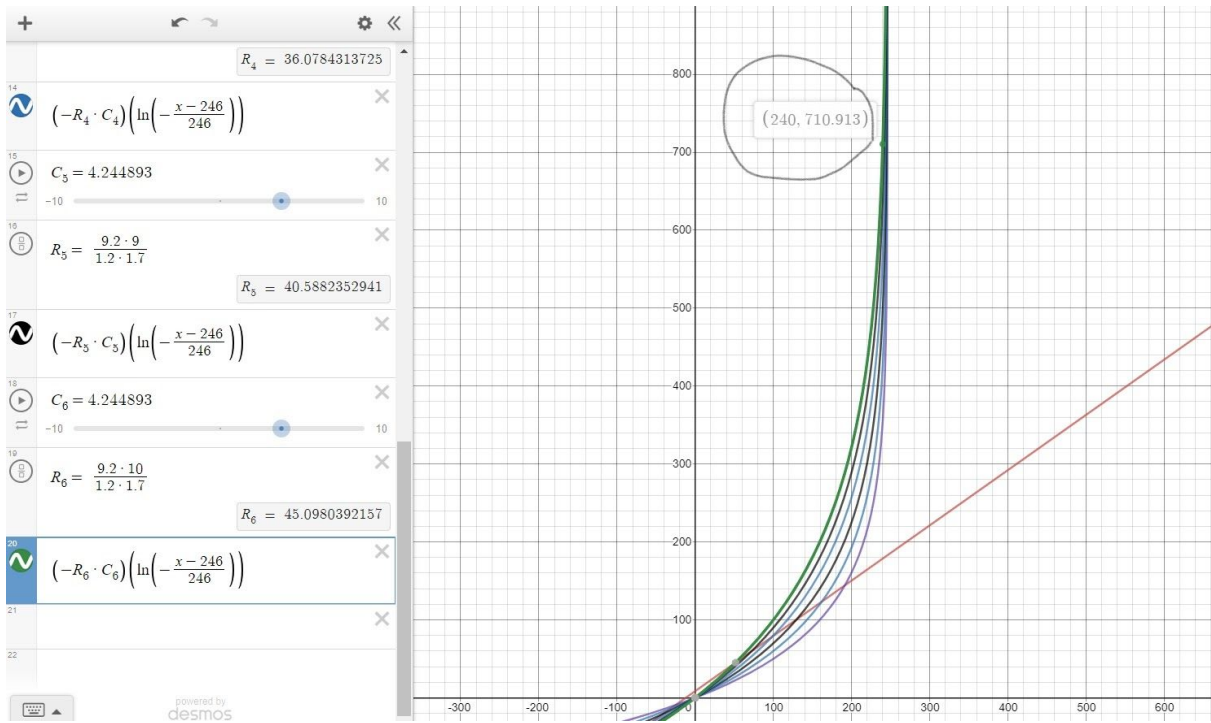




The last green colored function which is the function represents people at the -10th floor (6th floor of the park lot) , increases with the smallest slope since it has the most stair resistance because of this R

formula:  $\frac{9.2 \times n}{1.2 \times 1.695}$  where n is the shopping mall's floor number. So we should only consider

the last green colored function since when the last several people were ready to evacuate from the building, the people at -5th floor, -6th floor, -7th floor, -8th floor and -9th floor of the park lot would have already evacuated the building. Because of this we only considered the needed time for the last green colored function which is the function of the -10th floor (last floor of the park lot). However, all of these logarithmic functions had a vertical asymptote of 246. We had to make a wise decision on choosing the right last person in order to have accurate results from the graph. When the values pass the people number passes 240 the needed time gets unrealistically bigger because of the function properties of logarithmic functions. By considering this, we tried to choose the biggest people number as possible in order to be accurate. We selected the 240th person as a reference. By considering 240th person as our reference point, we found out the necessary time to evacuate the shopping mall for approximately 711 seconds. **(Figure 1.8)**



The result for park lot  $\approx 711$  seconds  $\approx 11.85$  minutes  $\approx 11$  minutes, 51 seconds

## CONCLUSION

Under the light of our mathematical modelling, we concluded that the residence evacuated within 7 minutes and 25 seconds, which is the total time needed to evacuate the total people above the ground. On the other hand, for the underground evacuation, we measured 2 hours and 26 minutes for the evacuation of the shopping mall; while, we measured 11 minutes and 51 seconds for the underground parking lot. Therefore, as a conclusion, the approximate time for the total evacuation of people in Sapphire is calculated as 2 hours and 38 minutes by adding the total time needed to evacuate the people in shopping mall and the total time needed to evacuate the people in parking lot, as the people in parking lot will exit the building after all people in the shopping mall exit.

According to our research, the building would collapse in 2 to 2 and a half hours due to the fire (*ÇAKICI YÜKSEK BİNALARDA ACİL BOŞALTIM SÜRESİNİN BELİRLENMESİ* ). Since the evacuation is estimated to take a longer time, we can come to the conclusion that the escape route of Sapphire is not safe enough to complete the evacuation before it is too late. Therefore, we propose that refining the capacity, escape route and plans of the building should be considered by Sapphire. Furthermore, as mentioned in the assumptions, the blueprints of Sapphire suggest that a person on one side of a resident floor can't reach the stairway on the opposite side. Thus, we propose that the hallways of the residence should be adjusted so that people from both sides have access to both stairways.

## ***REFLECTION***

### **Strengths**

With this modelling project we as a group were able to hone in on the subjects we were learning at school as well as learn subjects ahead of our school schedule. The method we used to calculate the total distance one person covered was based on our knowledge of arithmetic sequences. Reflecting our previous knowledge on this project not only helped us with our approach but also see the subject in a real life situation. As well as using previous knowledge, we also used a topic that we had yet to cover in class: integrals. By learning a subject foreign to us for the sake of our project we had the opportunity to develop our self-organization skills. Another thing that helped us understand how we will approach the modelling process was our physics knowledge. By using the idea of an electric current to express the stream of people evacuating the building we were able to have a better understanding of how our formula should end up looking like, thus creating interdisciplinary bonds.

Another strength we have about our model is how effectively we used technology for our advantage. We used coding as a way of simulating the real life actions and movements of the people

that would be evacuating the building. With this simulation, we also had the chance of cross-checking the data points that we gathered as a result of our many trials and the graph that was the product of our formula, which we found out to have high correlation. The double-checking made us more confident with our results.

For a big portion of our assumptions we relied heavily on architectural knowledge. In order to have a professional perspective on approaching the modelling process, we had meetings with architects about their professional approach to evacuation modeling, how to read floor plans and also used papers provided by them as resources all throughout our process.

## **Limitations**

Throughout our research, assumptions have shaped our methodology and conclusion. Since assumptions in a research have shortcomings which are considered as limitations, these limitations have restricted the application of practical knowledge on our mathematical model; therefore, our mathematical model is composed of theoretical knowledge and evidence based assumptions. However, the impact of these limitations have not greatly influenced the formulation of data, but the next person should consider these limitations and strive to eliminate them on their own research.

- ***Limitations about the building***

1) The floor plans of the Sapphire tower indicate the number of beds within that floor for the residence. Therefore, considering it as our assumption, we can predict the theoretical number of people within the floors which have their own floor plans. Under the light of our assumption, which is “According to the number of beds in the floor plans of the residence (the larger beds were counted as two people and the smaller beds were counted as one person), the average number of people to live on each floor was estimated to be 22.”, the number of total people in the tower, only for residence, is concluded as 1232. This assumption can be considered as the limitation of this project because there

can be more people than its expected in the floors, such as the guests, employees, and staff. Thus, the next person who will develop this project or use this project as reference to their project can use more practical data for the number of people in the floors of residence. Ultimately, that person can conclude more accurate conclusions which are more similar with the real life examples.

2) Furthermore, another limitation in our research is to take the total surface area's mean for assuming that the surface area of each apartment identically composes the total surface area. For instance, there are eight types of houses which all have different floor areas, and therefore the mean of the apartments' surface area will determine the dimensions of the building's floors. Therefore, as we measured the total number of people by referencing the Vitruvius Man, constructed by Leonardo Da Vinci, the total number of people is highly dependent on the surface area of the floors. Ultimately, the person who will improve or use our research project as a reference can accommodate the information of the surface area ratios between different types of apartments.

3) Lastly, we separated the types of apartments equally among the floors and obtained 6 floors which are not used as residence. Therefore, theoretically, we found 6 mechanical floors within the tower, but because of the lack of information about the distribution of the types of floors, we assumed that there would also be equal amounts of people within the mechanical floors. Thus, utilizing this assumption into our mathematical model, it can be considered as a limitation to our research. Finally, the next person who will improve or use our project as a reference can gather the information of people within the mechanical floors to conclude more accurate and reliable information to draw their conclusions.

- ***Limitations about the formulation***

1) As we developed the origin of our logarithmic function, which is from the function of charge on a capacitor, we tried to interfere the data gathered from smaller values of evacuation with



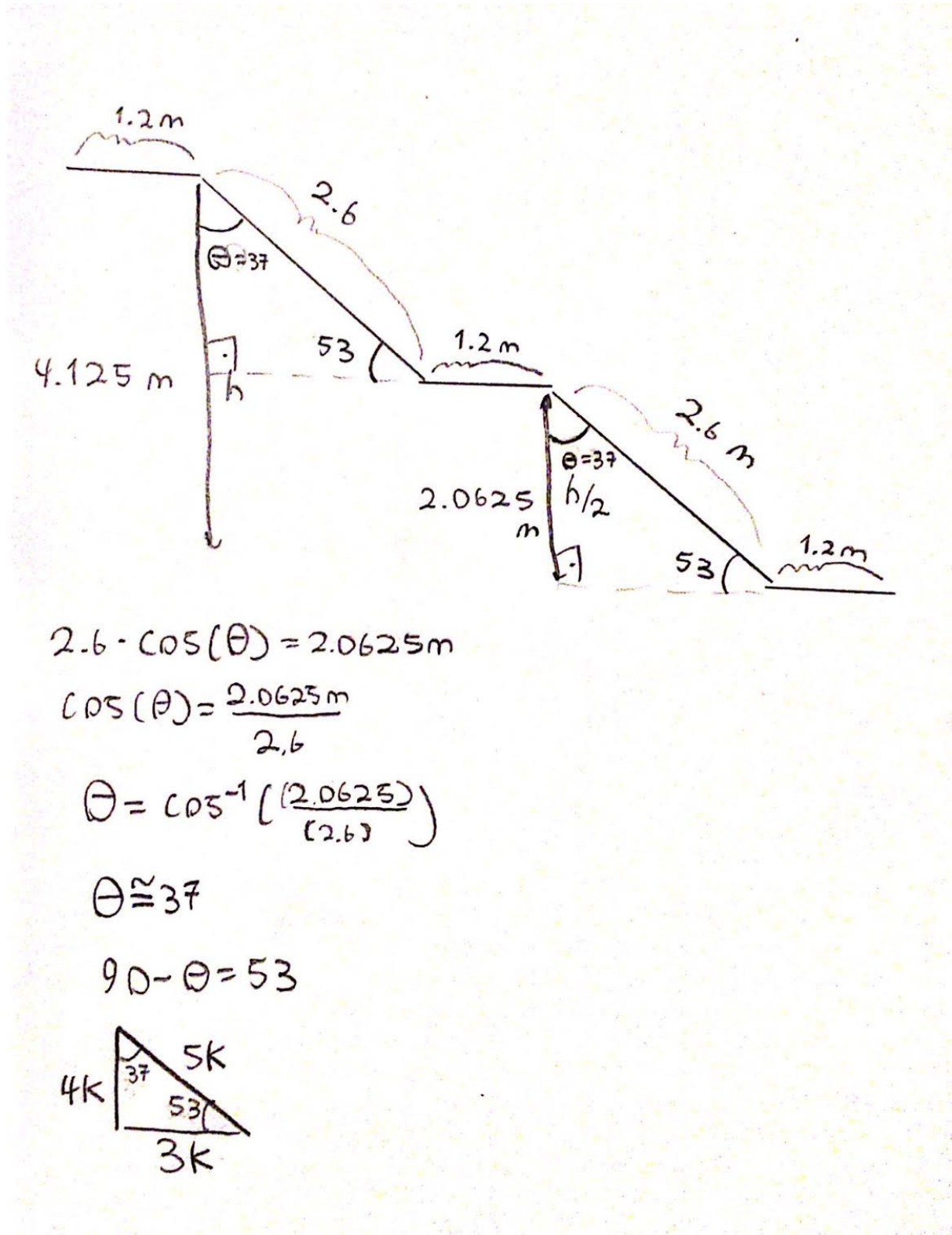
the variables in the function of charge on a capacitor to predict larger values for the evacuation of Sapphire. Because of the lack of 3-D technological opportunities to model thousands of people evacuating Sapphire, this method seemed efficient and applicable. However, to gather more accurate and reliable data from the modelled graph, the next person who will improve or use our research for reference can get in contact with advanced technological opportunities for modelling, which will decrease the margin of error by making the project more precise.

- 2) As our formula to find out the time taken to evacuate the shopping mall has originated from the capacitor charging formula, we used the variables in that formula to calculate the evacuation time. The variable “ $p$ ” that stands for resistivity in the parent formula of the parameter  $R$  was ignored in our formula because we disregarded the air resistance which people would face while evacuating Sapphire. If the air resistance was taken into account, there would have been a slight difference between the behaviors of these graphs. Therefore, the next person who will improve or reference our research can consider taking the variable “ $p$ ” into account for more accurate results and arrange their assumptions according to this situation.

## APPENDIX

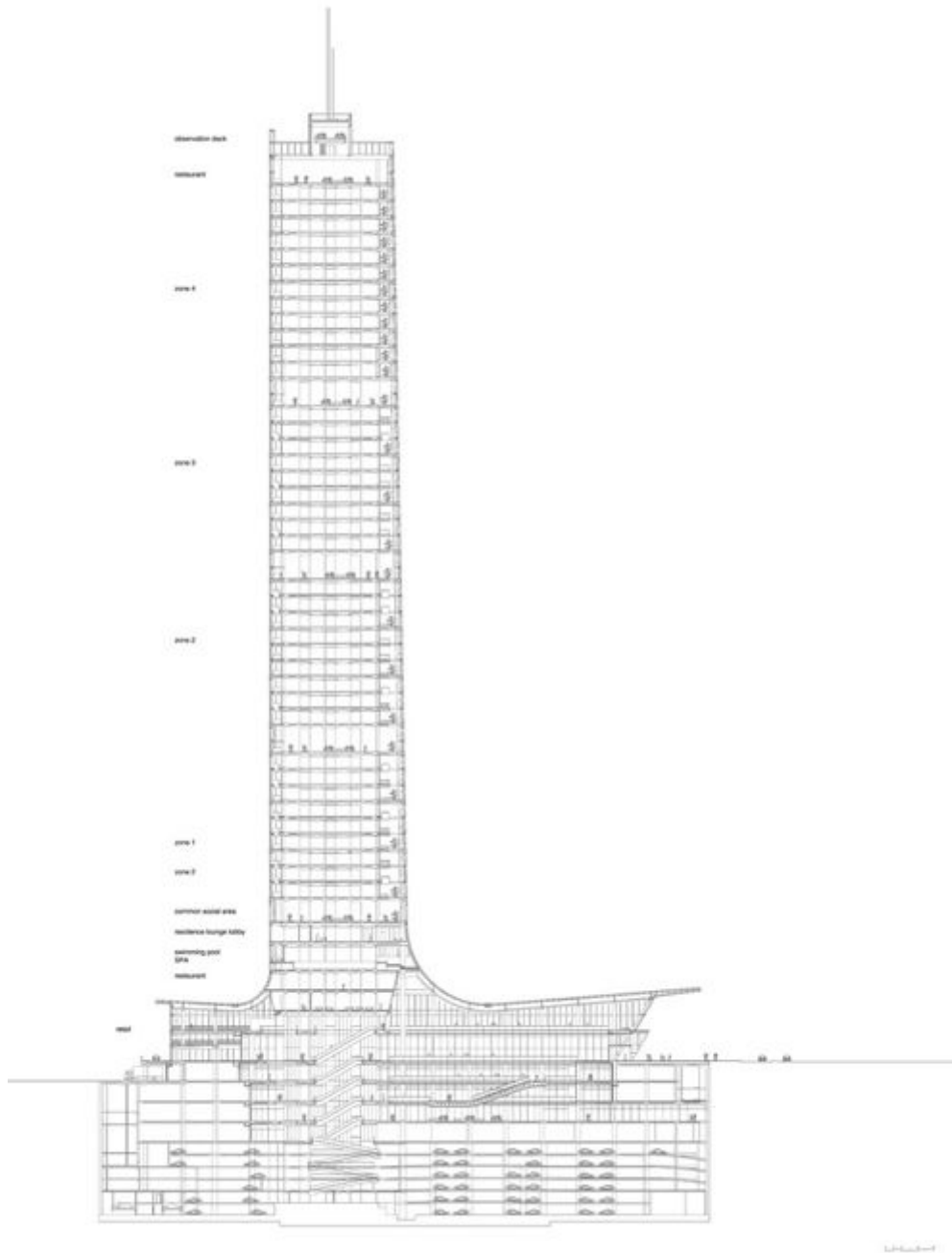
### Appendix 1.0:

This diagram shows the geometrical design of stairs and plates on a 2d plane.



## Appendix 1.1:

Blueprint of the building on a 2d plane.



**Appendix 1.2:**

Our finding on sapphire's dimensions on a 2d plane:

Step 1:  
 $10.8 \text{ cm} = 231 \text{ m}$   
 $1.50 \text{ cm} = 32.08 \text{ m}$

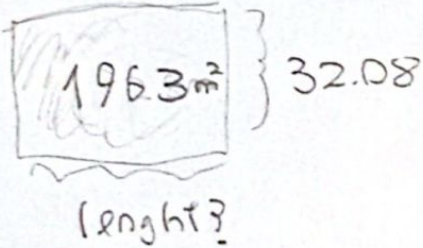
Step 2:  
Smallest residence =  $120 \text{ m}^2$   
biggest residence =  $1100 \text{ m}^2$

$\frac{120 \text{ m}^2 + 1100 \text{ m}^2}{2} = 610 \text{ m}^2$   
↓  
average area of all residence

Step 3:  
Total number of residence in Sapphire = 177

$177 \cdot 610 = 107970 \text{ m}^2$  → total area covered

Step 4:  
 $\frac{107970}{55} = 1963 \text{ m}^2$   
↓  
Total area covered Per Floor

Step 5:  
  
 $196.3 \text{ m}^2$  }  $32.08$   
length?

$\frac{1963}{32.08} \approx 61.19 \text{ m}$  length

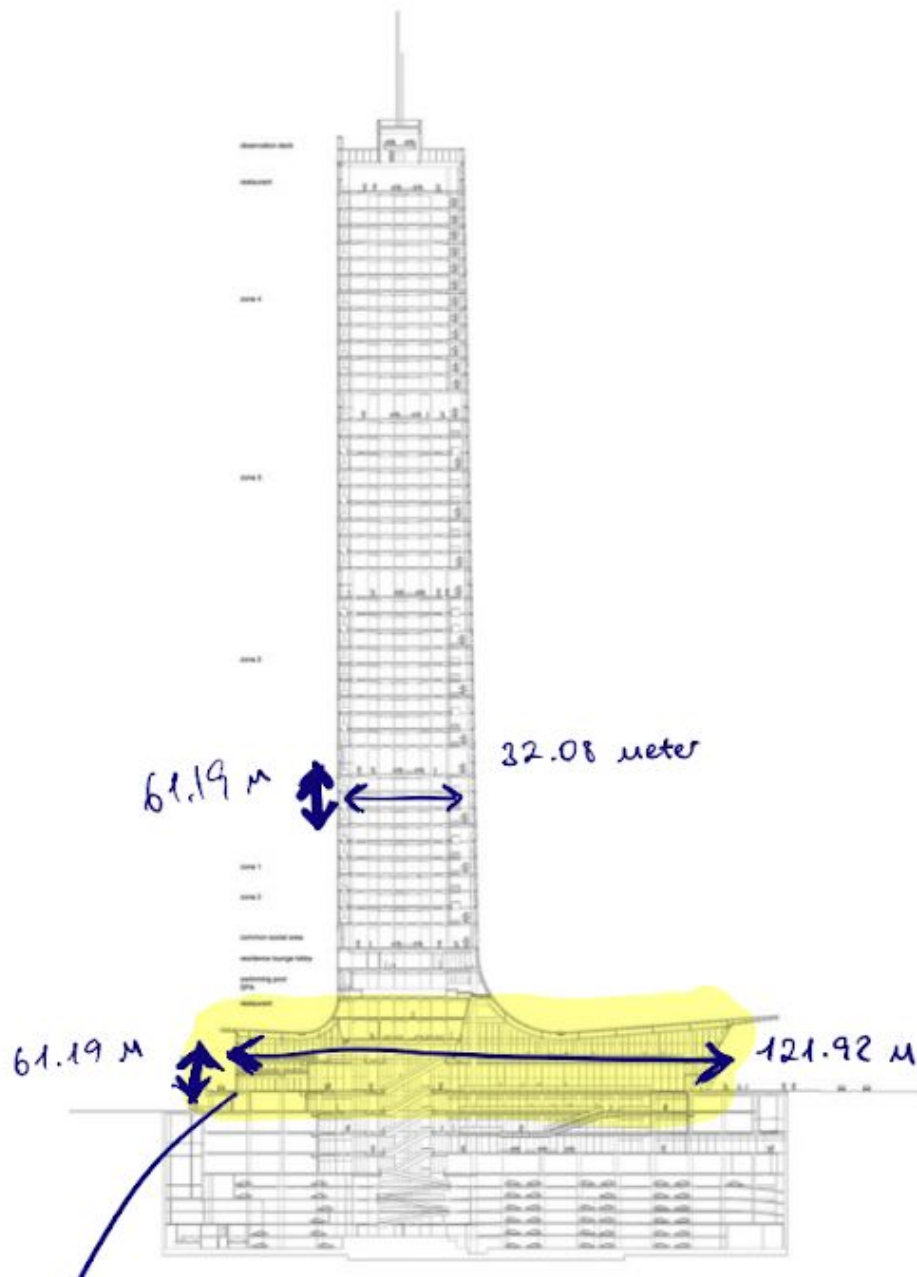
Diagram of a large rectangle with dimensions 61.19 m (width) and 231 m (height). A smaller rectangle is drawn inside it with dimensions 32.08 m (width) and 196.3 m (height).

Last step:  
231 m (height) meter  
32.08 (depth) meter  
61.19 (length) meter

CS CamScanner ile tarandı

### Appendix 1.3:

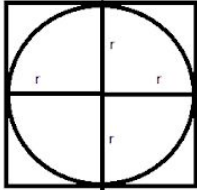
The dimensions of shopping mall:



## Appendix 1.4:

Calculating the people number in shopping mall by persons area and shopping malls area (not to scale!)

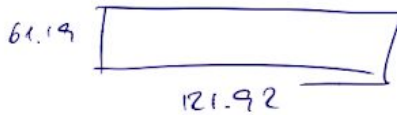
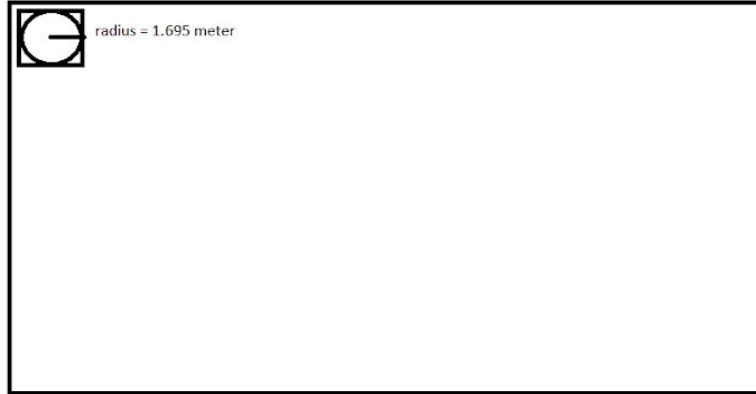
Geometry Part:



$2r = R = \text{height}$   
 $(2r)^2 = \text{height}^2 = \text{area}$   
covered by a single  
person

32.08 meter

61.19 meter



$$\frac{61.19 \cdot 121.92}{1.695 \cdot 1.695}$$

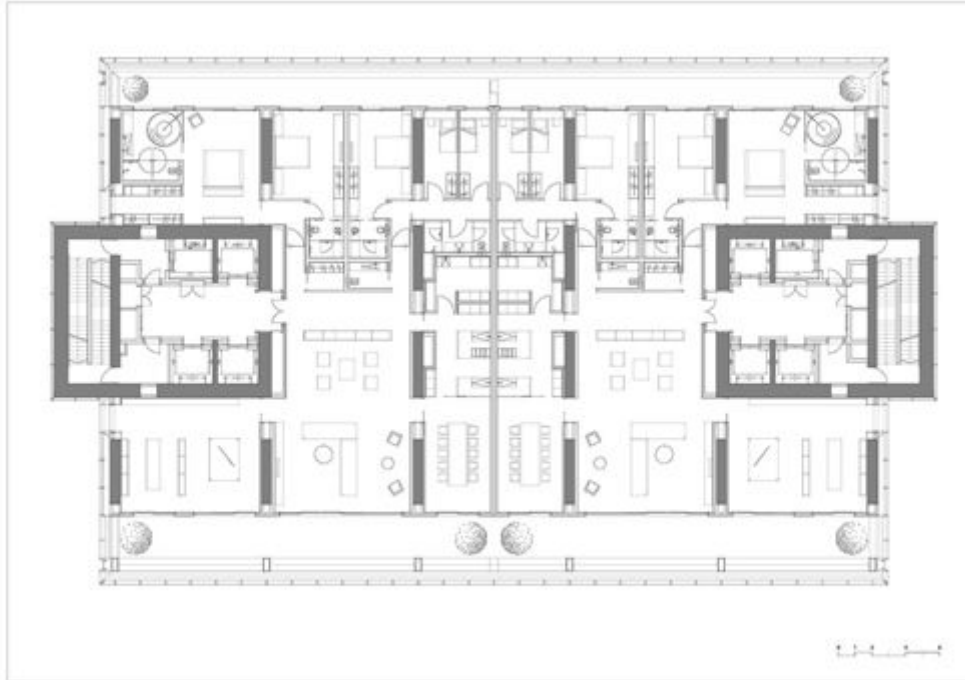
$\approx 2597$

her  
bir  
AVM  
katinde  
insan  
sayisi

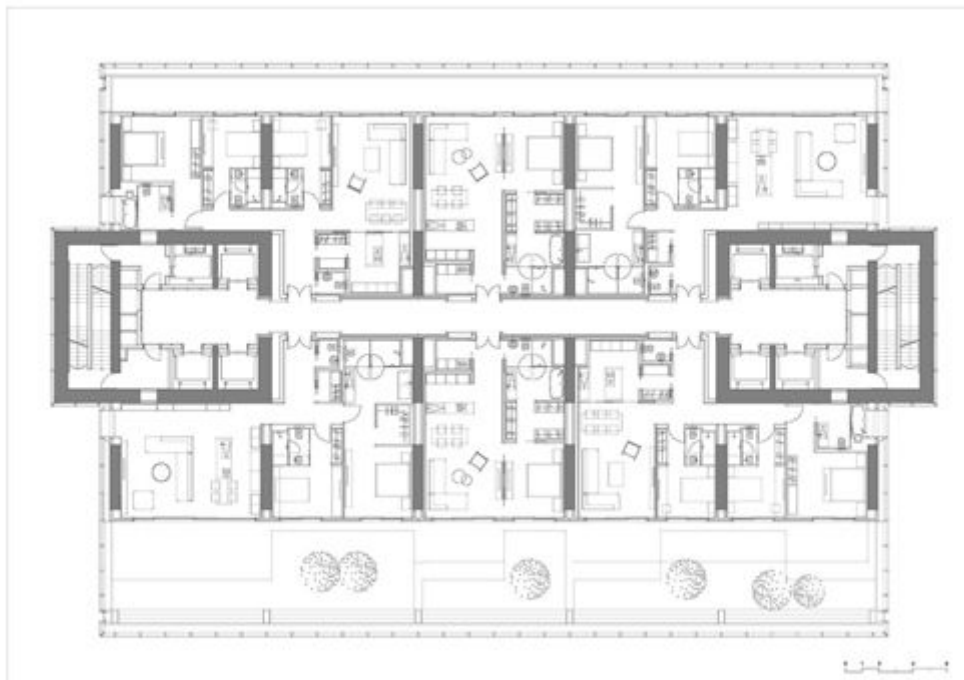
**Appendix 1.5:**

Some blueprints of apartment plans

**Appendix 1.5.0:**

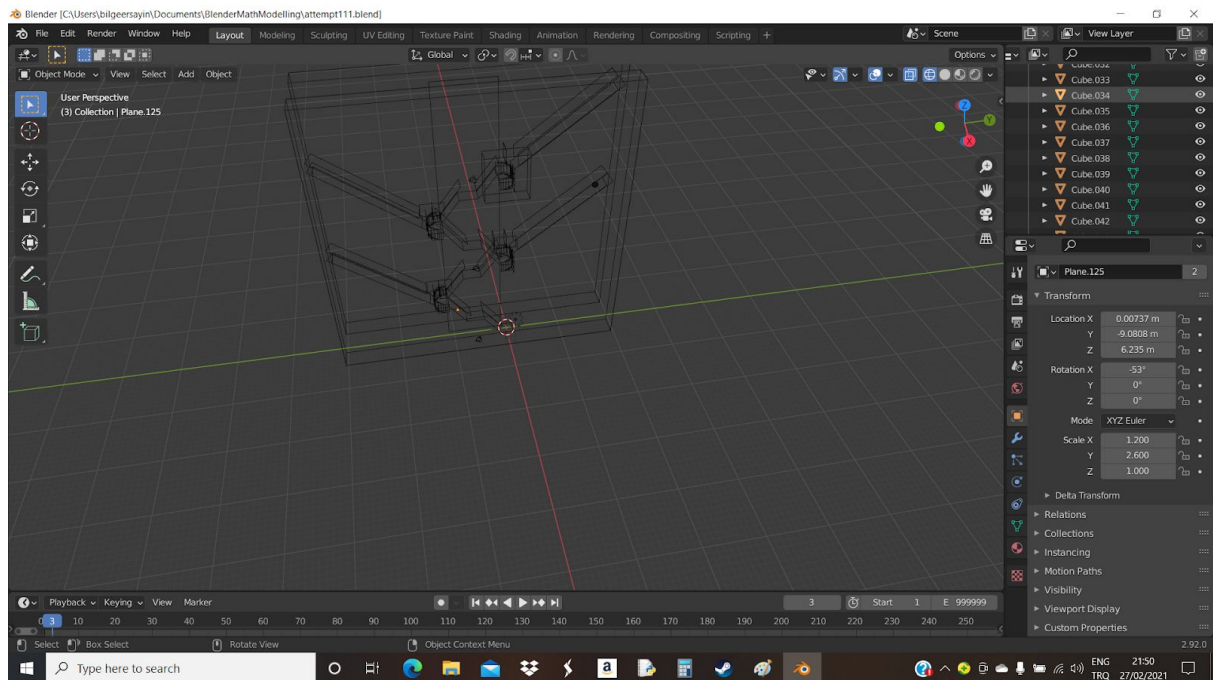


**Appendix 1.5.1:**



## Appendix 1.6:

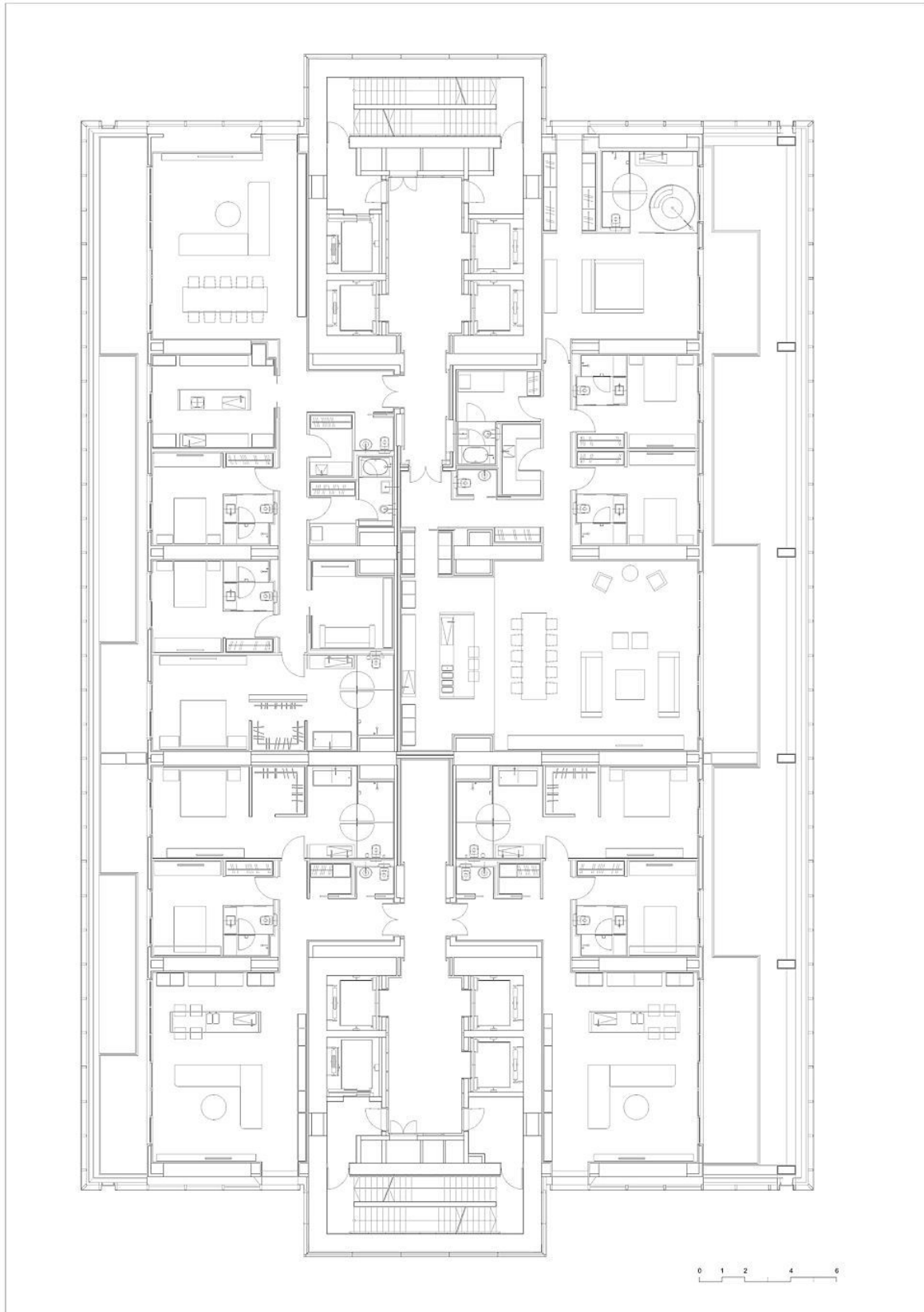
The visual look of the stairs that we designed.





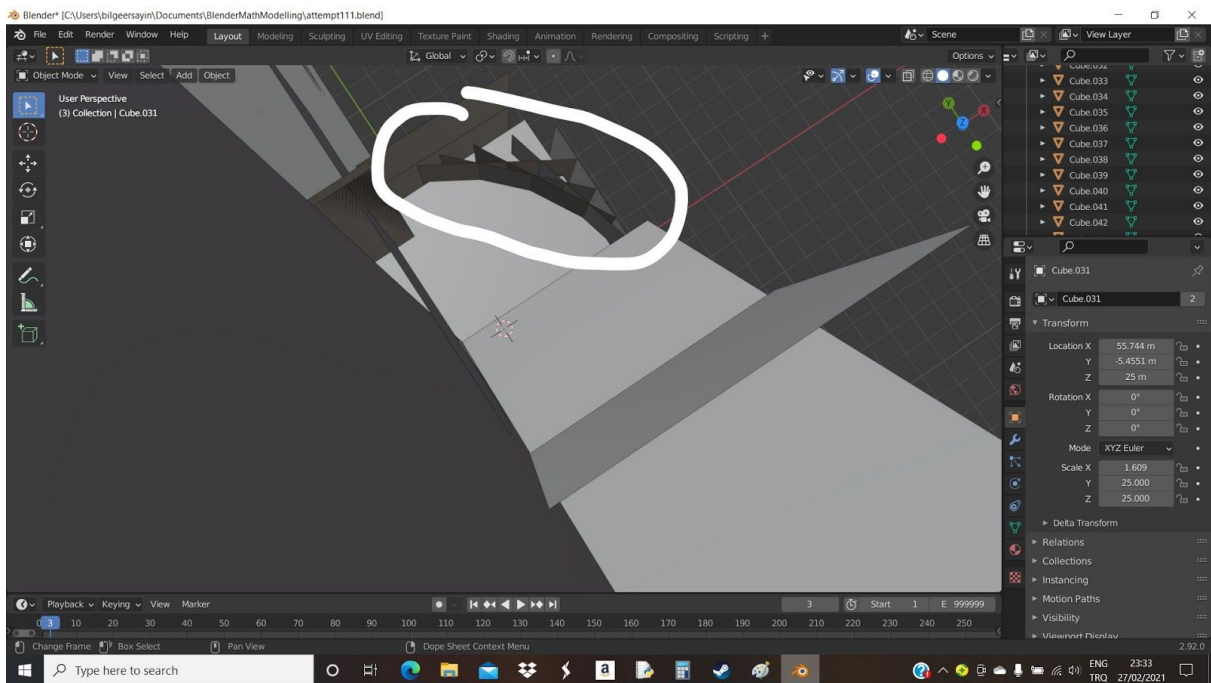
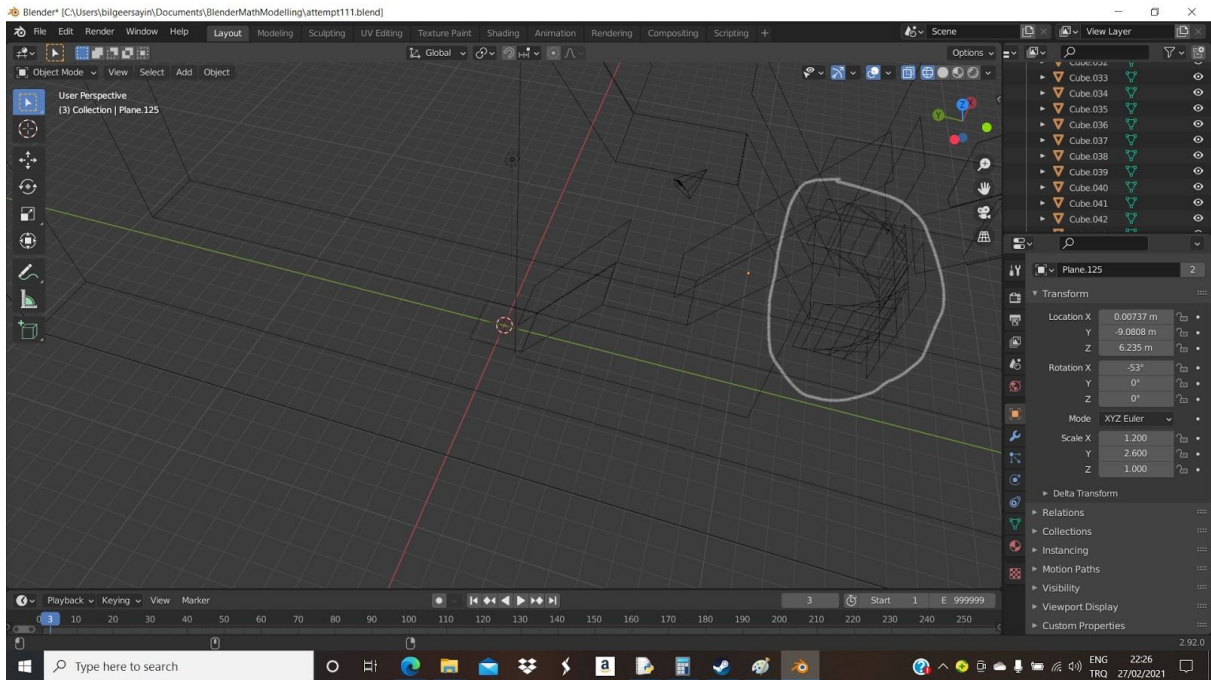
**Appendix 1.7:**

The blueprint of the building with scales



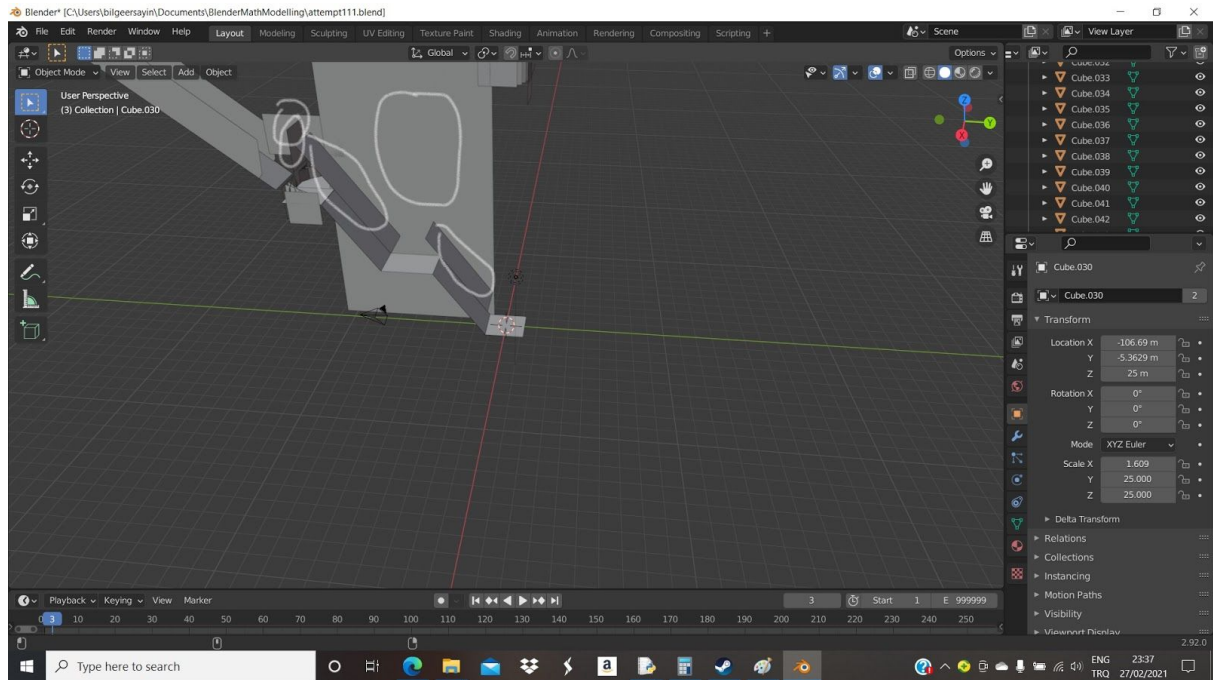
## Appendix 1.8:

The half circular shape created by forming up triangles whose degree decrease by 15 every step



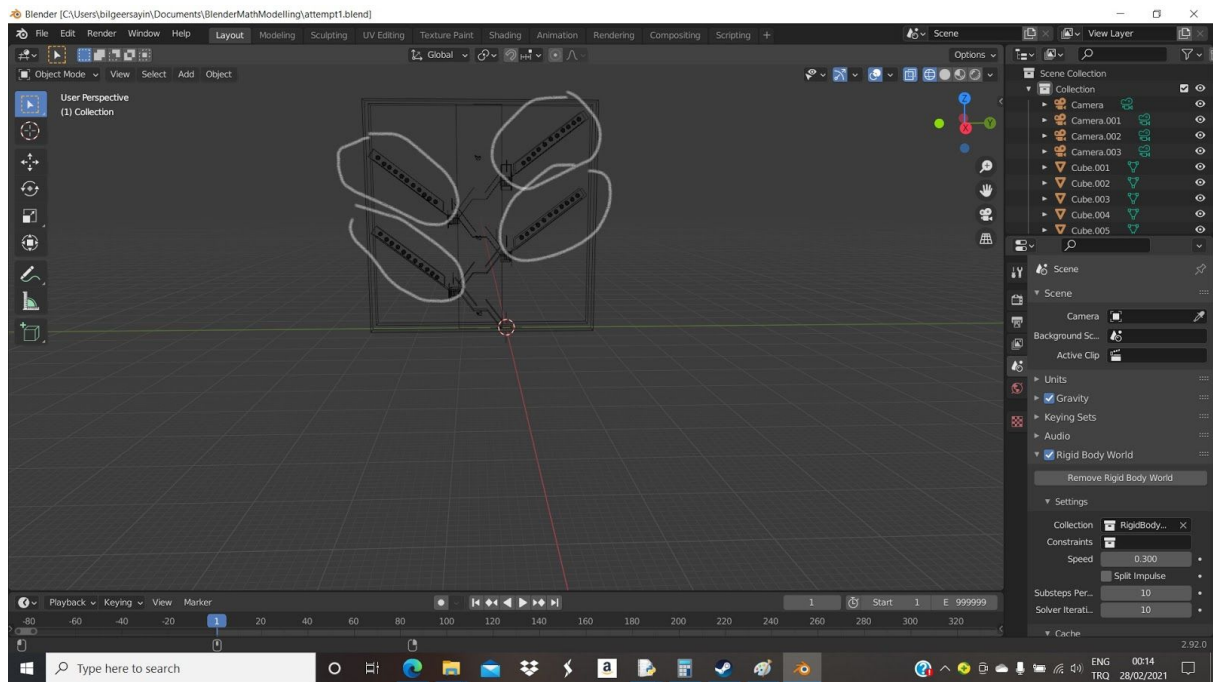
## Appendix 1.9:

The necessary bumpers that were used to avoid spheres to jump or bounce



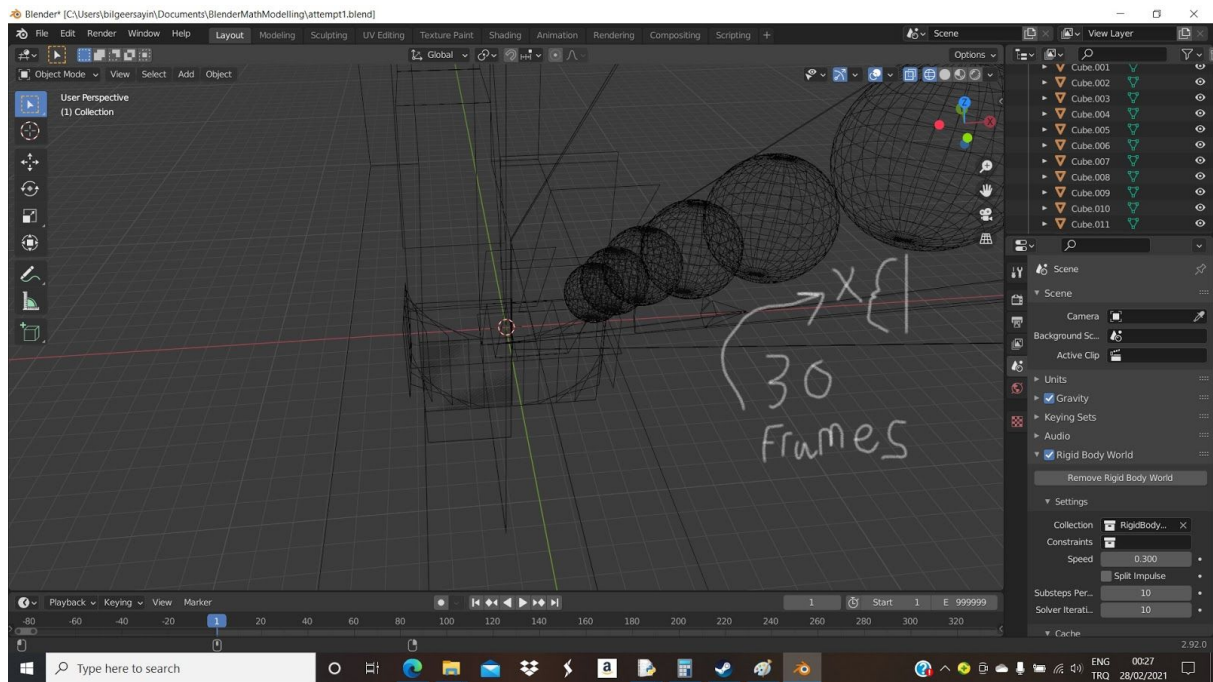
## Appendix 2.0:

The number of spheres in each floor



## Appendix 2.1:

The approximate time difference for laying on the stairways



## Appendix 2.2:

Data collection of spheres evacuating the stairs:

1<sup>st</sup> ball <sup>(first floor)</sup>  $\Rightarrow$  16.4 seconds

2<sup>nd</sup> ball <sup>(second floor)</sup>  $\Rightarrow$  17.2 seconds

3<sup>rd</sup> ball <sup>(third floor)</sup>  $\Rightarrow$  16.2 seconds

4<sup>th</sup> ball <sup>(fourth floor)</sup>  $\Rightarrow$  16.6 seconds

$$\frac{16.4 + 17.2 + 16.2 + 16.6}{4} = 16.6 \text{ seconds } \left. \vphantom{\frac{16.4 + 17.2 + 16.2 + 16.6}{4}} \right\} \text{ average}$$

$$\text{Average speed} = 1.4 \text{ m s}^{-1}$$

10m  $\Rightarrow$  stair way

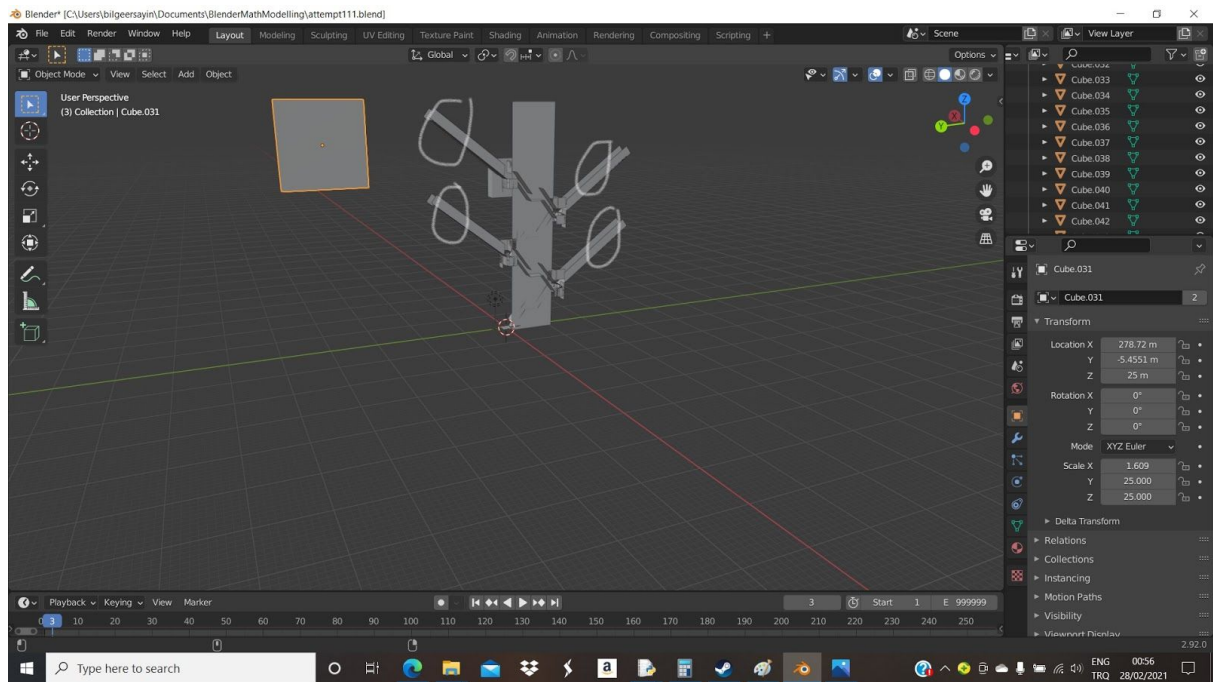
9.2m  $\Rightarrow$  the length of stairs that connect floors

$(9.2 + 10)\text{m}$  16.6 seconds  
 $9.2\text{m}$   $\times$  7.8 seconds  $\rightarrow$  needed to finish stairs

$(9.2 + 10)\text{m}$  16.6 seconds  
 $10\text{m}$   $\times$  8.5 seconds  $\rightarrow$  needed to finish stair ways

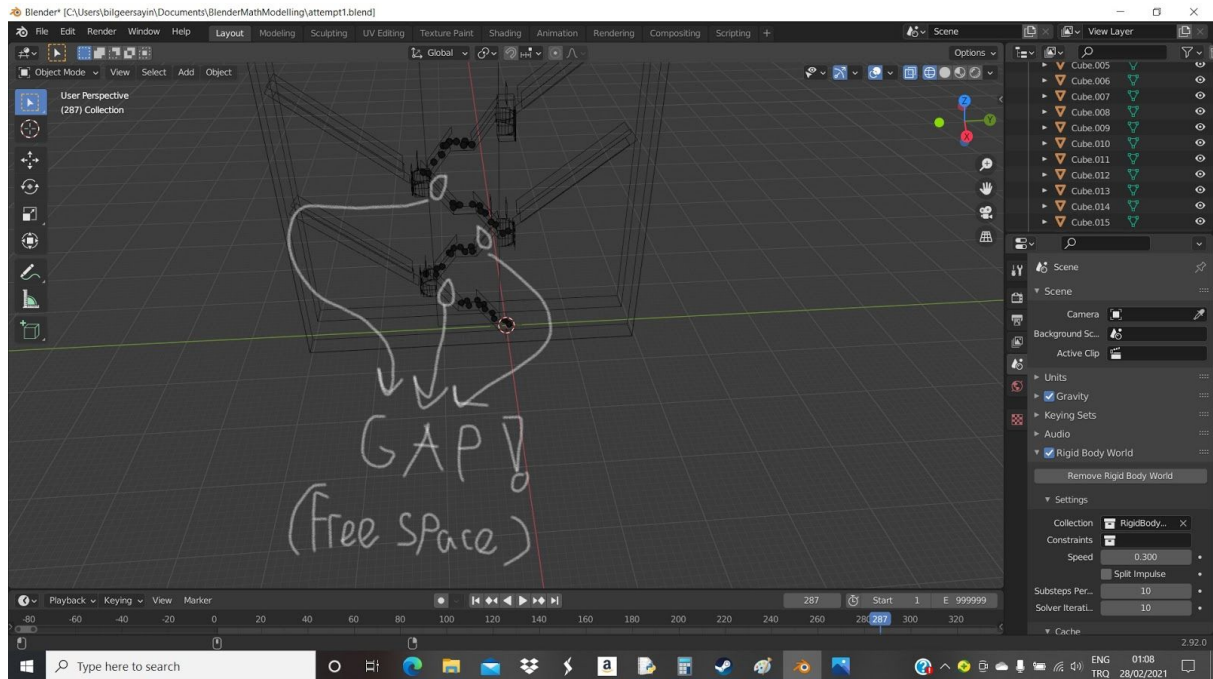
## Appendix 2.3:

The stairways that connect apartments and actual stairs



## Appendix 2.4:

The evidence of spheres that are located at different floors, do not meet or collide

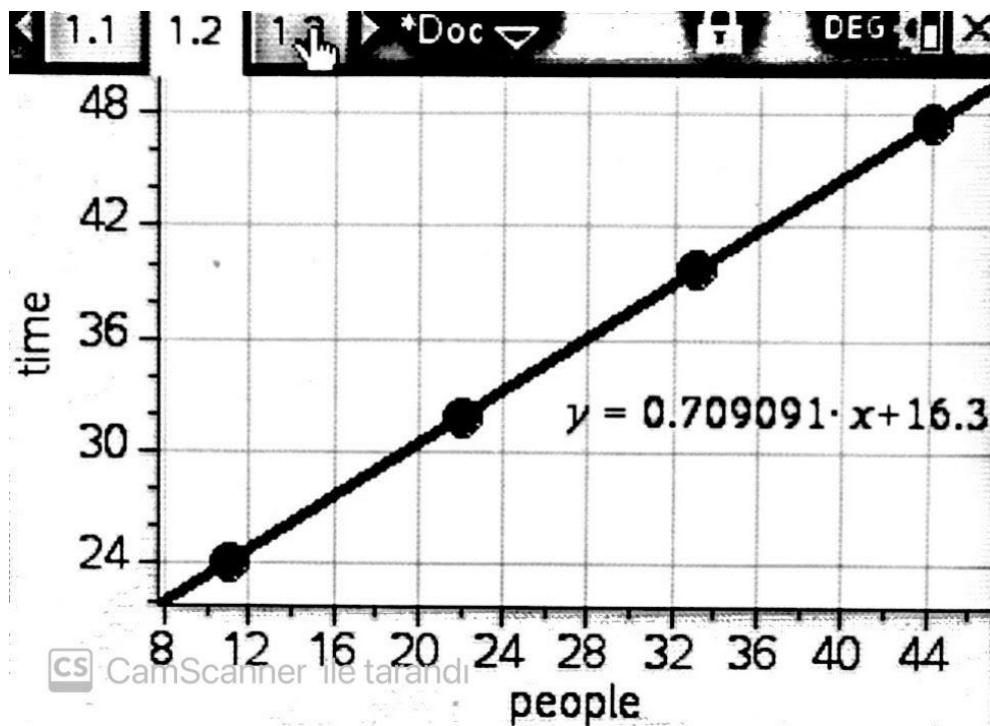




### Appendix 2.5:

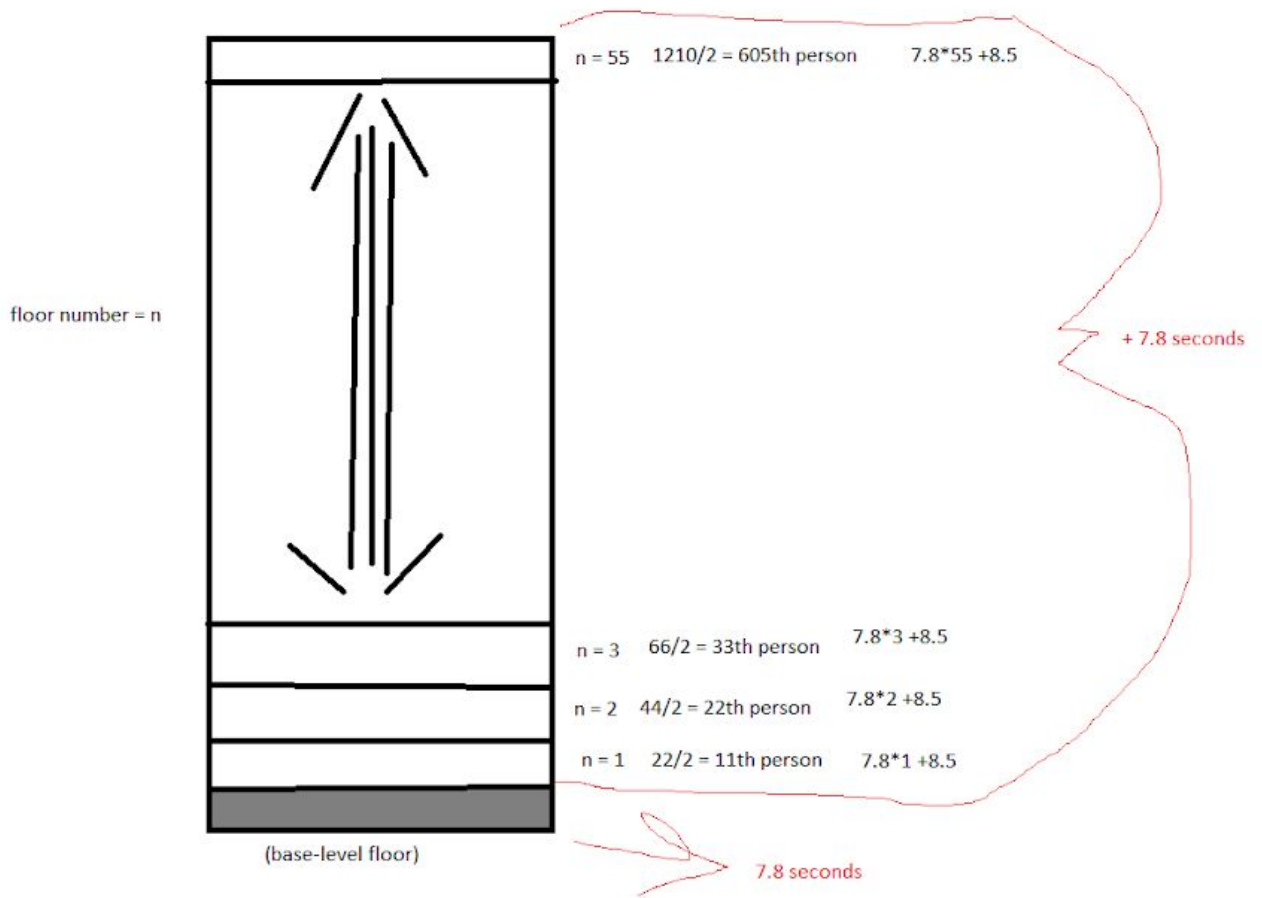
The graph of the residence (apartment) evacuating datas

	A people	B time	C	D
1	11.	$7.8 \cdot 1 + 8.5$		
2	22.	$7.8 \cdot 2 + 8.5$		
3	33.	$7.8 \cdot 3 + 8.5$		
4	44.	$7.8 \cdot 4 + 8.5$		
5				



**Appendix 2.6:**

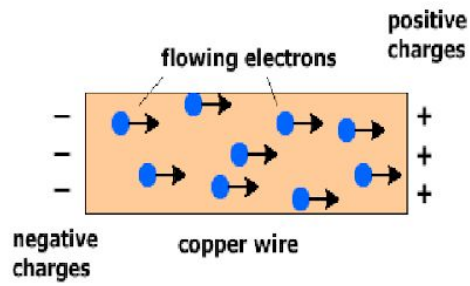
The 2d diagram that shows the time distribution sequence



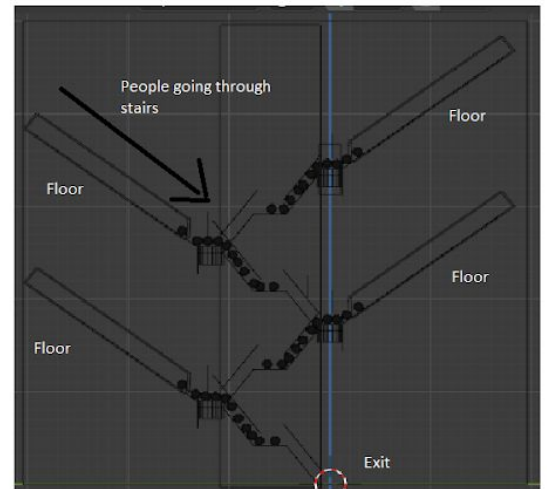
## Appendix 2.7:

The diagram that shows the similarities of electrons flowing through the wires and people running through stairs

Electronics :  
Electrons Running  
through a copper wire  
Goal: To reach the  
positive side (+) from  
the negative side (-)



Real Life:  
People running through  
stairs  
Goal: To reach the exit  
of the building from the  
floor number they have  
started



**Appendix 2.8:**

The Resistance formula:

$$R = \frac{\rho L}{A}$$

$\rho$  = resistivity

$L$  = length

$A$  = cross sectional area

## Appendix 2.9:

The relation between length of the wire and the resistance of the wire and the relation between the cross section area of the wire and the resistance of the wire

Example 1:

$$\begin{array}{l} \text{cross section area } 3.14 * r^2 \\ \text{length} = d \end{array} \quad \text{resistance value} = \frac{d}{3.14 * r^2} = R1$$

If we double the length of the wire:

$$\text{new resistance value} = \frac{2d}{3.14 * r^2} = R2 = R1 * 2$$

The second resistance value appears as the doubled value of the first resistance value

If we double the cross section area of the wire:

$$\text{new resistance value} = \frac{d}{2 * (3.14 * r^2)} = R3 = R1 / 2$$

The third resistance value appears as the halved value of the first resistance value

## Appendix 3.0:

The page informs us about the usage of capacitors and their functions

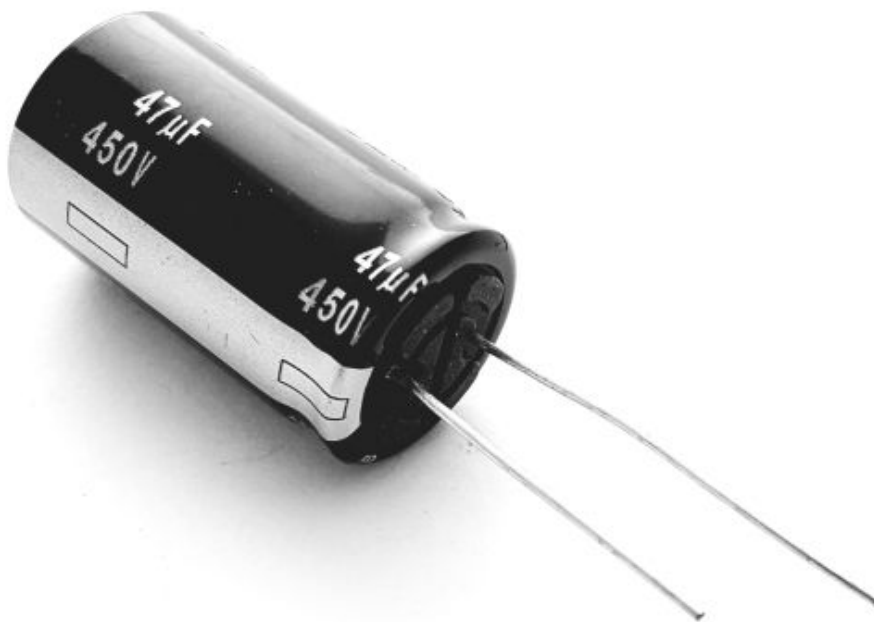
# What Is Capacitor?

by Richard

☰ Page Contents >

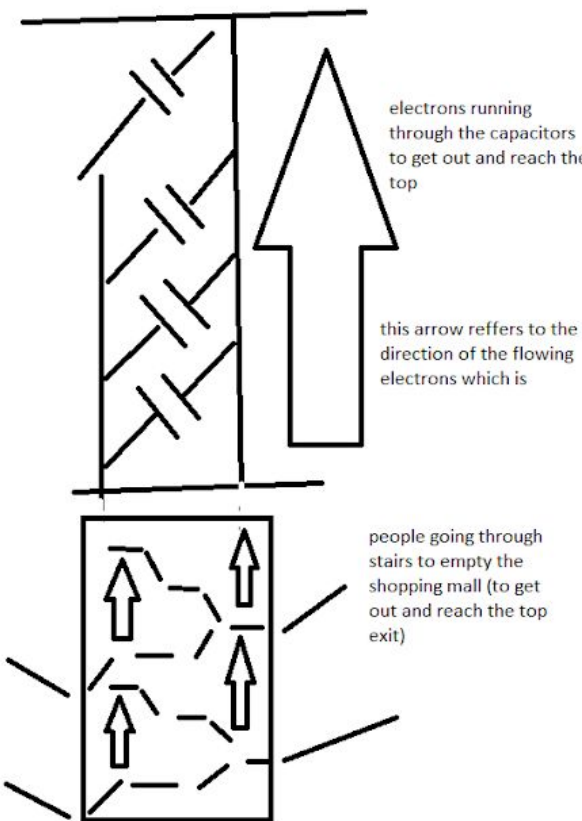
## What Is Capacitors?

A capacitor, also known as a condenser, is a device that stores energy in an electrical field. Capacitors are open circuits to DC current, but their [impedance](#) drops as AC current passing through them has its frequency increased. Extremely high-frequency currents see capacitors as short circuits.



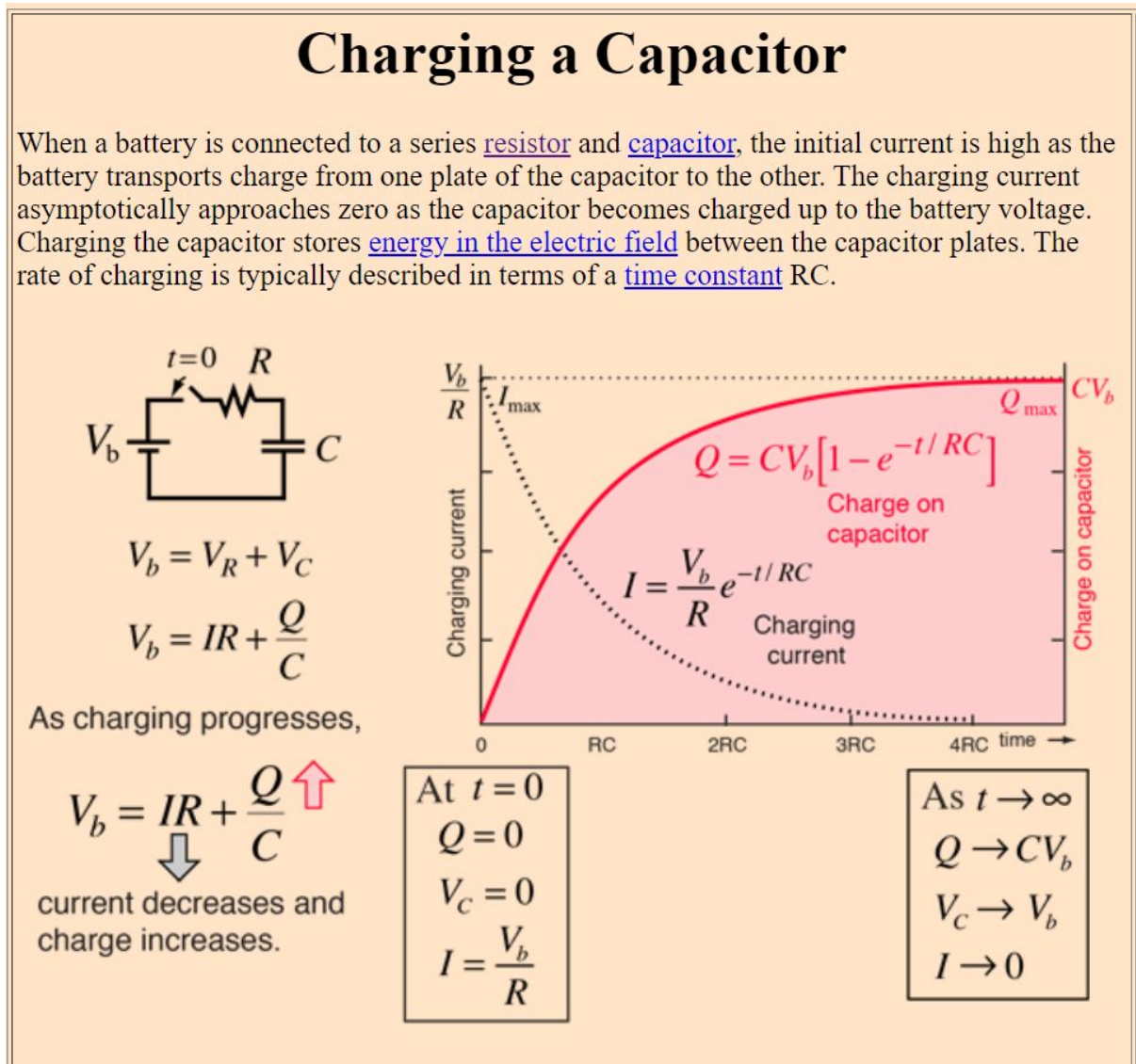
### Appendix 3.1:

The diagram that shows the relationship and the similarities between capacitor charging function and evaluating progress

Electronic/Physics	In real life evacuation of a building
 <p>electrons running through the capacitors to get out and reach the top</p> <p>this arrow refers to the direction of the flowing electrons which is</p> <p>people going through stairs to empty the shopping mall (to get out and reach the top exit)</p>	<p>Q = The number of people that ran away from the building by going through stairs ■ till that unit time</p> <p>CVb = The total number of people that were going to empty the building. (Population of people in the building)</p> <p>e = constant</p> <p>"1 -" stand for the function transformation rule</p> <p>t stands for the unit time that is taken by the people (unit: seconds)</p> <p>the minus sign ( - ) that is in front of "t" is needed because function transformation rules</p> <p>R stands for the total resistance of the stairs (stair length / stair cross section area)</p> <p>No corresponding value for C in real life. The value C would be obtained at on going steps</p>
<p>Q = The number of electrons that ran way from the capacitor ■ till that unit time</p> <p>CVb = The total number of electrons (charges) that were going to flow through capacitor</p> <p>e = constant</p> <p>"1 -" stand for the function transformation rule</p> <p>t stands for the unit time that is taken by the electrons (unit: seconds)</p> <p>the minus sign ( - ) that is in front of "t" is needed because function transformation rules</p> <p>R stands for the reistance of the circuit</p> <p>C value stands for the capacitance of the capacitor</p>	<p>Q = The number of people that ran away from the building by going through stairs ■ till that unit time</p> <p>CVb = The total number of people that were going to empty the building. (Population of people in the building)</p> <p>e = constant</p> <p>"1 -" stand for the function transformation rule</p> <p>t stands for the unit time that is taken by the people (unit: seconds)</p> <p>the minus sign ( - ) that is in front of "t" is needed because function transformation rules</p> <p>R stands for the total resistance of the stairs (stair length / stair cross section area)</p> <p>No corresponding value for C in real life. The value C would be obtained at on going steps</p>

## Appendix 3.2

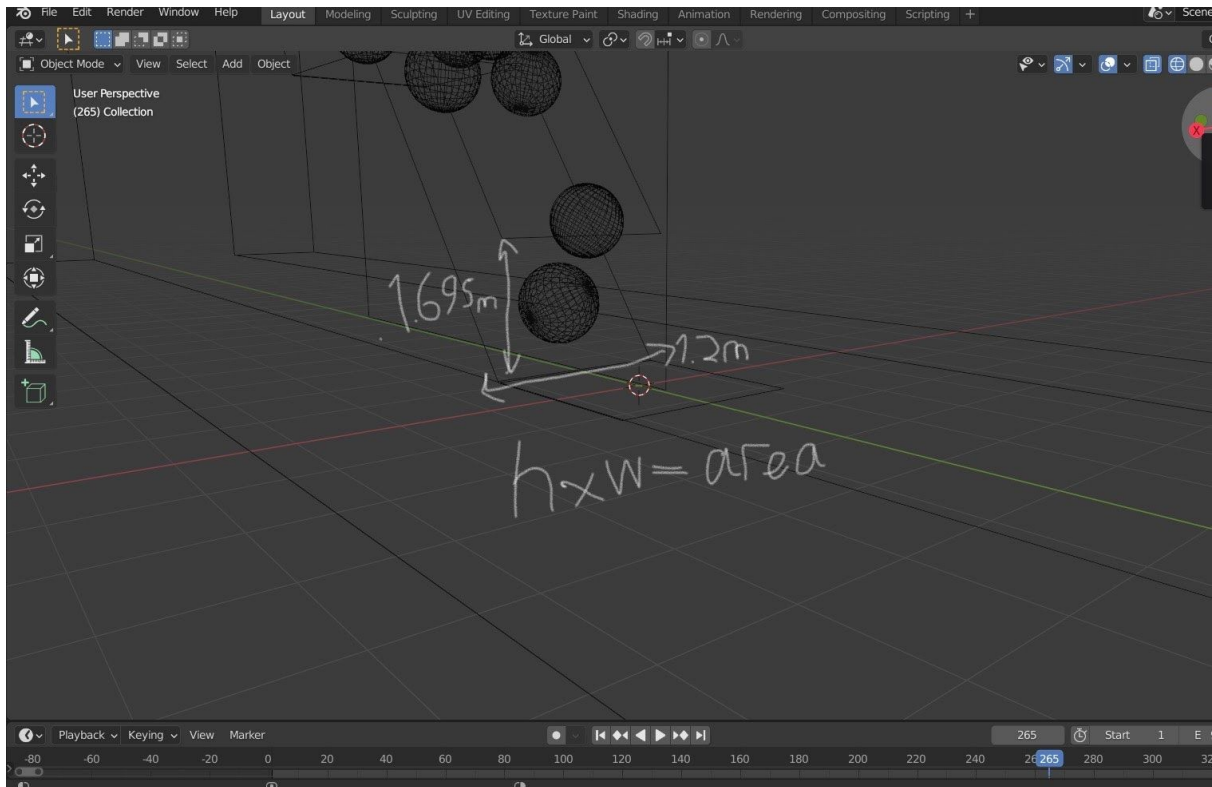
Capacitor charging function:





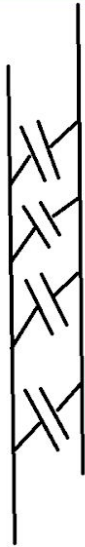
### Appendix 3.3:

The look of the cross section area of the stairs



### Appendix 3.4 :

The diagram and formula that shows us the effect resistance on the formula and evacuation time



$$T = R \cdot C //$$

$$Q = Q_1 + Q_2 + Q_3 + Q_4$$

$$= Q(1 - e^{-t/T_1}) + Q(1 - e^{-t/T_2}) + Q(1 - e^{-t/T_3})$$

$$\text{Since } R_4 > R_3 > R_2 > R_1, \quad + Q(1 - e^{-t/T_4})$$

$$T_4 > T_3 > T_2 > T_1 //$$

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